

# Beam-beam interactions and their compensation in RHIC and LHC

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# Outline

- Motivation
- Beam-Beam Simulation Code (BBSIMC)
- Beam-beam and beam-wire interactions at RHIC
- Beam-beam simulation for wire compensation at LHC
- Electron lens at RHIC

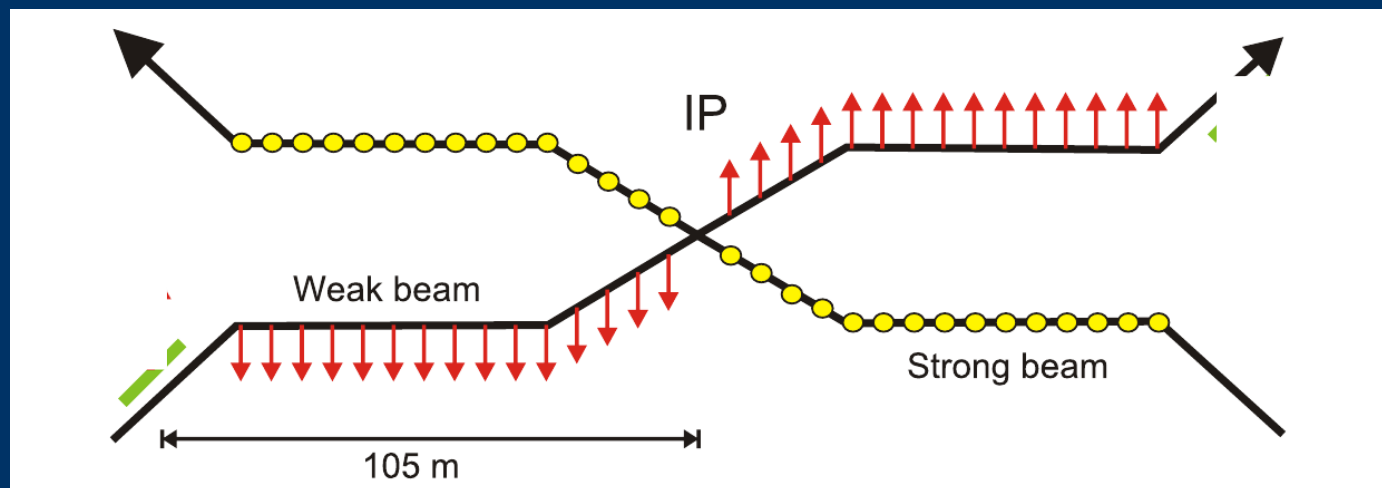
Phys. Rev. ST Accel. Beams 12, 031001 (2009)  
PAC'09 WE6PFP031 and WE6PFP032

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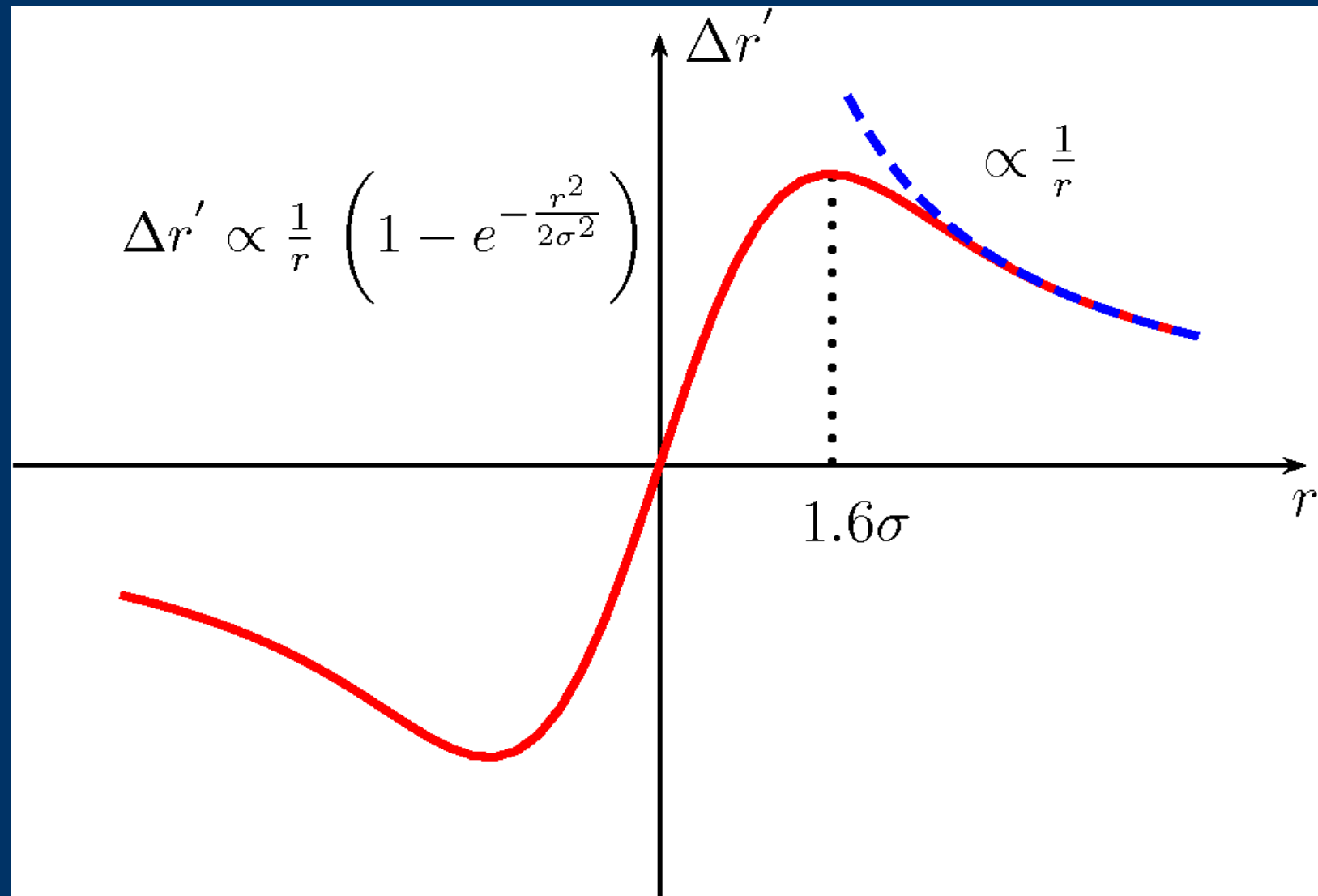
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## Beam-beam interactions

- One of major sources which cause emittance growth or beam loss.
- Head-on at IPs and long-range at parasitic crossings.
- Expected to deteriorate beam quality in LHC, because of large beam intensity ( $1.2E11$ ) and many bunches (30 parasitic crossings per IP).
- Need ways to reduce the effects:
  - Electron lens for head-on beam-beam compensation
  - Current carrying wire for long-range beam-beam compensation



## Beam-beam force



# Current carrying wire for long-range collision

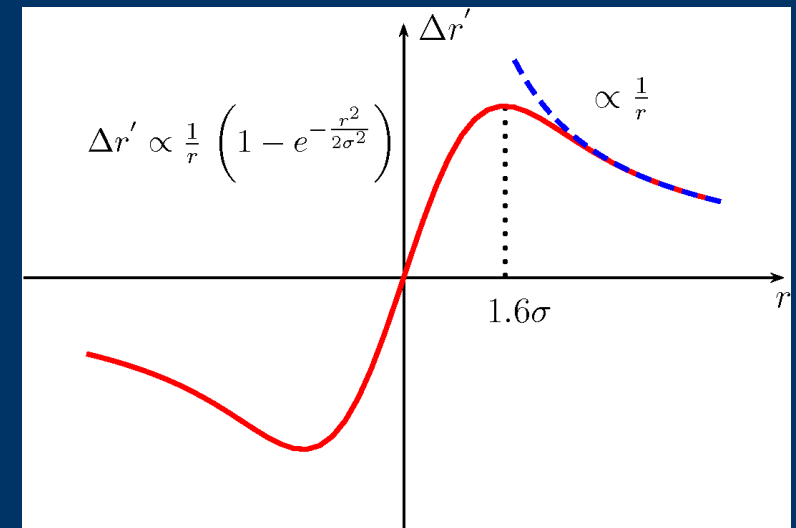
- For a large separation distance at parasitic crossings, the strength of long-range interaction is inversely proportional to the distance.
- Its effect on a test beam can be compensated by current carrying wires which create just the same field.
- The advantage of such an approach consists of the simplicity of the method and the possibility to deal with all multipole orders at once.

- Beam-beam kick of round beam

$$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \frac{2N_b r_0}{\gamma_b} \frac{1}{r^2} \left( 1 - \exp \left[ -\frac{r^2}{2\sigma_b^2} \right] \right) \begin{pmatrix} x \\ y \end{pmatrix}$$

- Wire kick

$$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \frac{\mu_0 (IL)_w}{2\pi (B\rho)} \frac{1}{r^2} \begin{pmatrix} x \\ y \end{pmatrix}$$



## Low energy electron lens for head-on collision

- Low energy electron beam which is matched to a profile of high energy colliding beam acts as a defocusing or focusing lens which compensates effect of the colliding beam.
- Beam-beam kick of round beam

$$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \frac{2N_b r_0}{\gamma_b} \frac{1}{r^2} \left( 1 - \exp \left[ -\frac{r^2}{2\sigma_b^2} \right] \right) \begin{pmatrix} x \\ y \end{pmatrix}$$

- Elens kick of Gaussian electron beam

$$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \frac{2N_e r_0}{\gamma_b} \frac{1}{r^2} \left( 1 - \exp \left[ -\frac{r^2}{2\sigma_e^2} \right] \right) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} \longrightarrow \quad N_e &= N_{IP} \cdot N_b \\ \sigma_b &= \sigma_e \end{aligned}$$

## Motivation

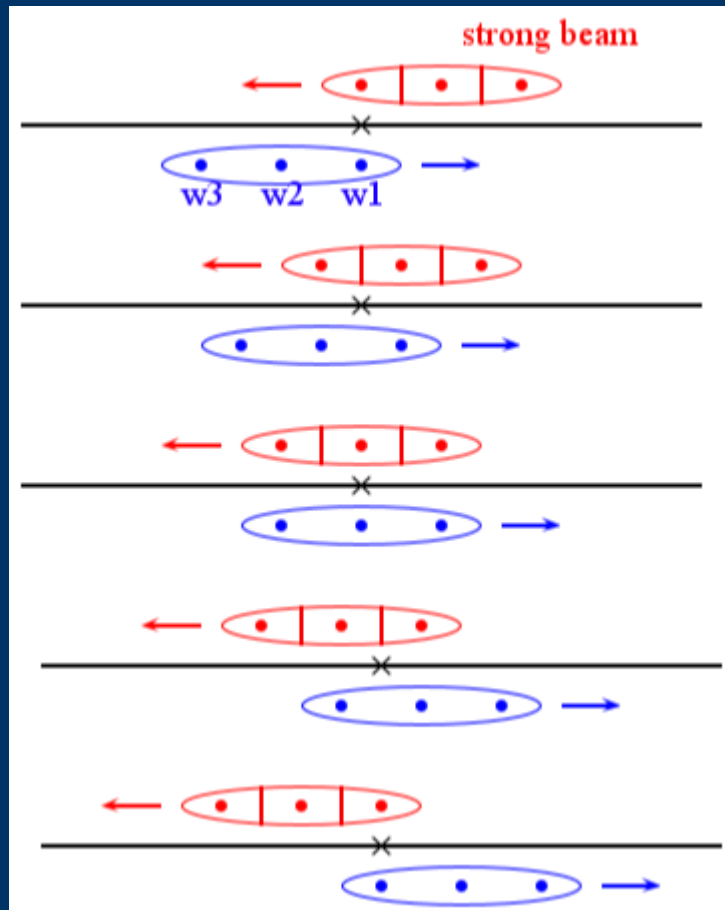
- In LHC, both head-on and long-range interactions are an issue due to large beam intensity and many bunches.
  - Wire compensation will be tested in RHIC as a proof of principle.
  - RHIC is also interested in head-on compensation with an electron lens to mitigate emittance growth.
  - Wire compensator: installed in 2006.
  - Electron lens: will be installed by end of 2011.
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## Beam-Beam Simulation Code (BBSIMC)

- 6D weak-strong tracking code
  - Linear transfer matrices btwn nonlinear elements + nonlinear kicks at the nonlinear elements (thin lens approximation: sextupoles, multipoles, etc.)
  - Beam-beam force: (1) Gaussian beam profile and (2) Poisson solver with FFT.
  - Multiple-slice model for finite bunch length effects
  - Lorentz boost to handle crossing angle collisions
  - Modules: wire and electron lens compensation, BTF, and diffusion
  - Fully parallelized with MPI.
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## Multiple slice model for head-on



- The strong bunch is divided into slices in a longitudinal direction to consider the finite bunch length effect of the beam-beam interaction.
- In the simulations, we applied 11 slices in the main IPs where the beta function is comparable with the bunch length.
- Each slice in a beam interacts with particles in the other beam in turn at the collision points.

# Beam-Beam Force

- Bassetti-Erskine formula for elliptic Gaussian beam profile

$$\Delta x' = \frac{2\tilde{n}_* r_0}{\gamma} \frac{\sqrt{\pi}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \Im W(x, y)$$

$$\Delta y' = \frac{2\tilde{n}_* r_0}{\gamma} \frac{\sqrt{\pi}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \Re W(x, y)$$

$$W(x, y) = w\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) - e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}} w\left(\frac{\frac{x\sigma_y}{\sigma_x} + i\frac{y\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right)$$

, where  $n^*$  is number of particle per bunch,  $r_0$  is classical radius of particle,  $\gamma$  is Lorentz factor, and  $w$  is complex error function.

## Beam-Beam Force

- Poisson solver with FFT for arbitrary beam profile
- Green function solution of Poisson equation

$$\phi(\vec{r}) = \int G(\vec{r}, \vec{r}') \rho(\vec{r}') d\vec{r}'$$

$$G(x, y : x', y') = -\frac{1}{4\pi} \ln \left[ (x - x')^2 + (y - y')^2 \right]$$

Using convolution theorem and inverse Fourier transform, one can get

$$\phi(\vec{r}) = \mathcal{F}^{-1} \left( \hat{G}(\vec{\omega}) \hat{\rho}(\vec{\omega}) \right)$$

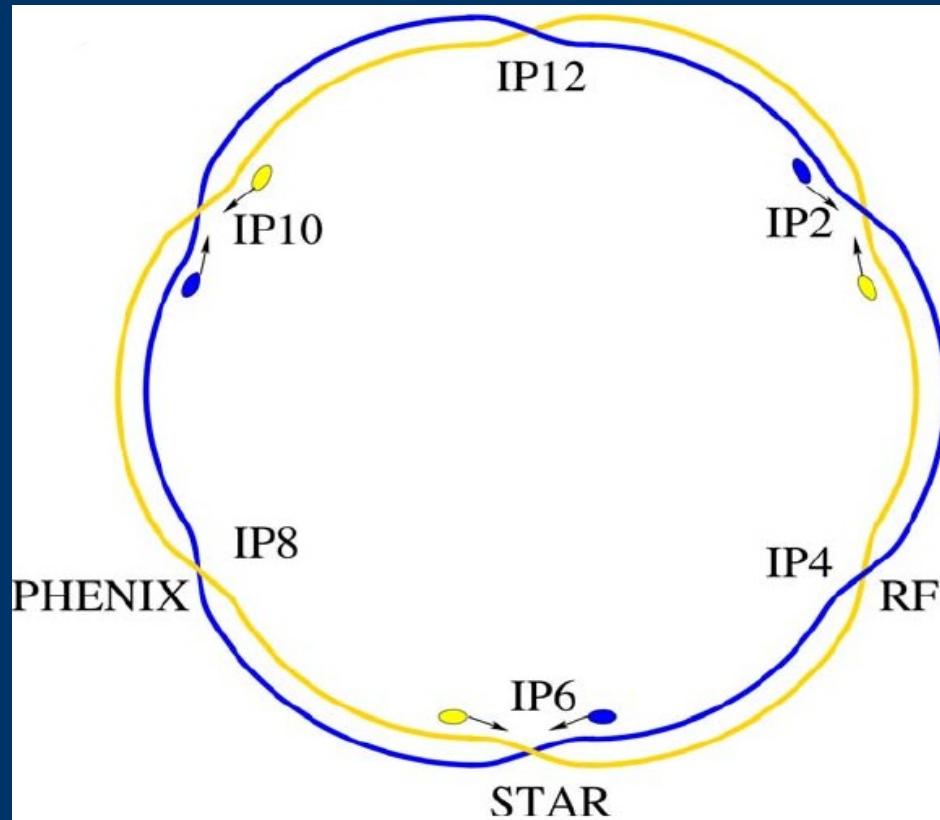
$$\hat{G}(\vec{\omega}) = \left( \frac{1}{\sqrt{\pi}} \right)^2 \int_{\mathbb{R}^2} G(\vec{r}) e^{-i\vec{\omega} \cdot \vec{r}} d\vec{r}$$

$$\hat{\rho}(\vec{\omega}) = \left( \frac{1}{\sqrt{\pi}} \right)^2 \int_{\mathbb{R}^2} \rho(\vec{r}) e^{-i\vec{\omega} \cdot \vec{r}} d\vec{r}$$

**Beam-beam and beam-wire interactions at  
RHIC**



# *RHIC (Relativistic Heavy Ion Collider)*



- RHIC is used as a test bed for a wire compensator.
- Head-on collisions at IP6/8.
- In this study, simulate only Blue beam.

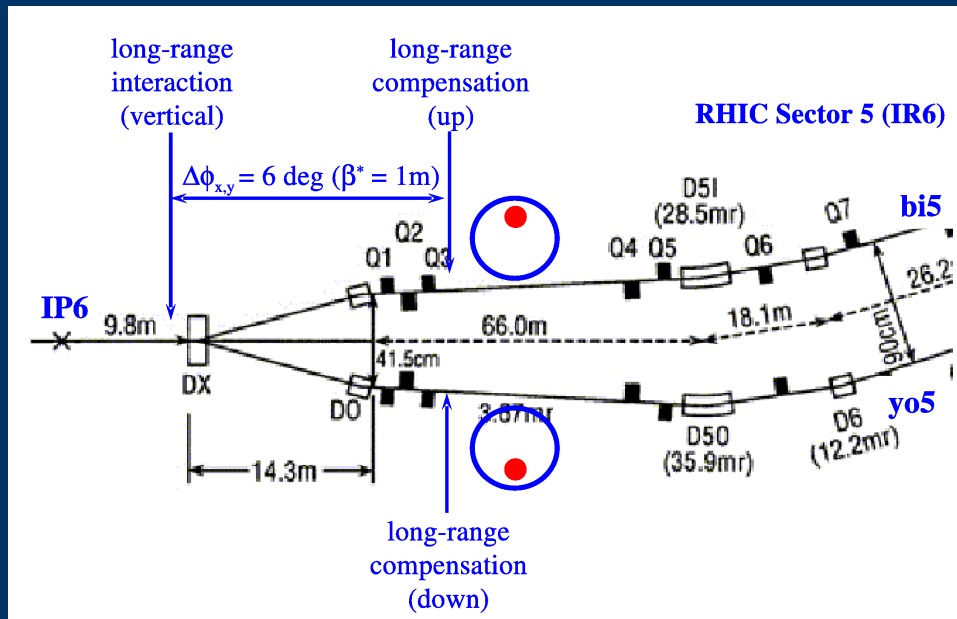
# *RHIC Parameters*

	unit	Gold beam	Deuteron beam
Energy	<i>Gev/n</i>	100	107
Bunch intensity	1E9	1	134
Emittance (95%)	mm-mrad	18	18
Beta* at IP6	m	1	0.9
Beta(x,y) at wire location	m	(1100,390)	(1200,400)
Beam-beam parameter	1E-3	1.3	1.5
Nominal tune		(0.220, 0.231)	(0.235, 0.225)
Chromaticity		+2	+2

- Gold beam: gold(Blue)+gold(Yellow)
- Deuteron Beam: deuteron(Blue)+gold(Yellow)

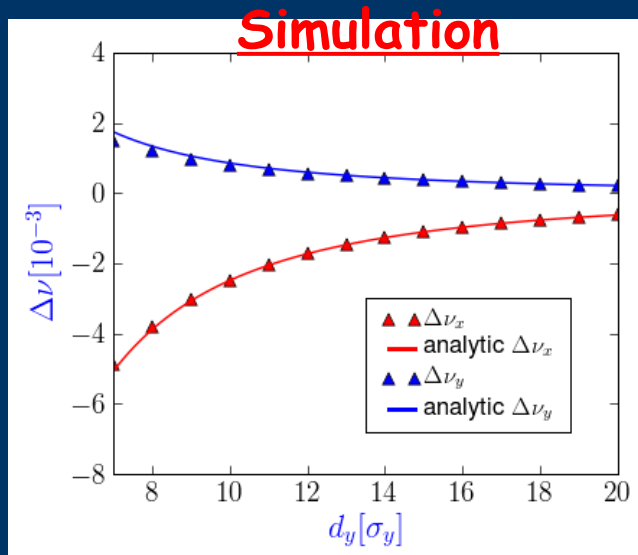
# Wire compensator in RHIC

- Two wires are installed (one for each beam).
- Phase advance betwn DX magnet and wire location is 5.7 degree.
- To compensate a single long-range, the current strength (IL) is required by  $(IL) = Nb * q * c$ , (Nb=bunch intensity, q=charge, c=speed of light).
- $(IL) = 3.8 \text{ A-m}$  (for Gold beam),  $6.5 \text{ A-m}$  (for Deuteron beam).
- Maximum wire strength is  $125 \text{ A-m}$  (Max. current is 50A).



- To see the effect of wire, max. current is applied in the experiment and simulation.

# Tune: wire position scan (RHIC)

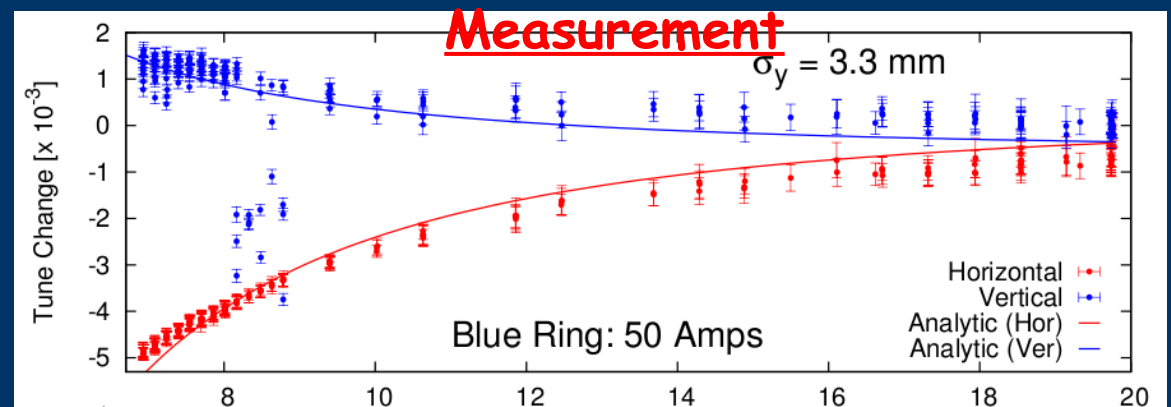


- The full lines are the curves calculated using the following expression:

$$\Delta\nu_{x,y} = \pm \frac{\mu_0 I_w L_w}{8\pi^2 (B\rho) \sigma^2} \beta_{x,y} \frac{d_y^2 - d_x^2}{(d_y^2 + d_x^2)^2}$$

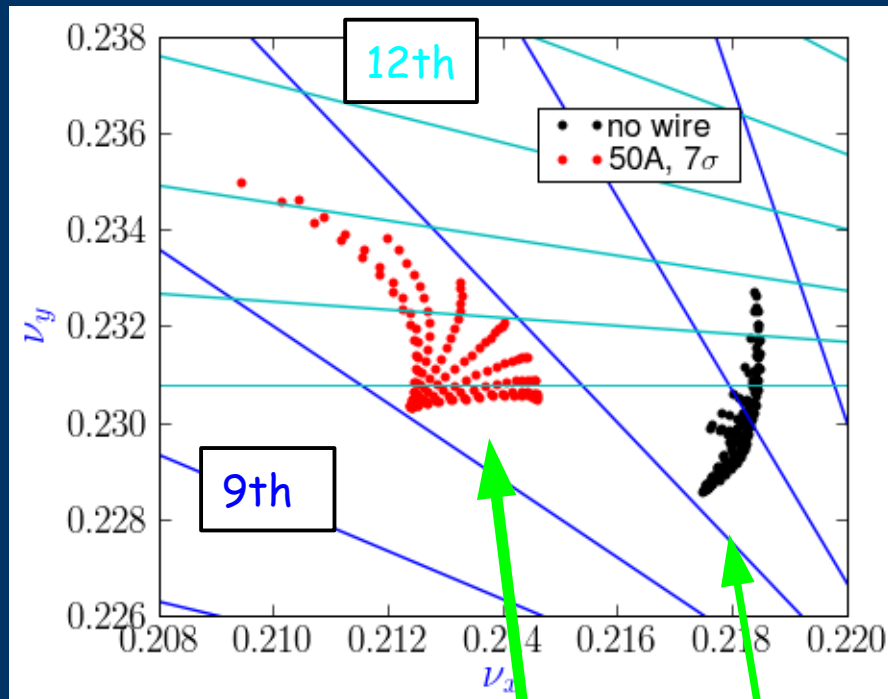
- Measurements and simulations also agree.

- Data sets are obtained at gold beam at store energy. (Abreu, Fisher)





## Gold beam: tune footprint (RHIC)

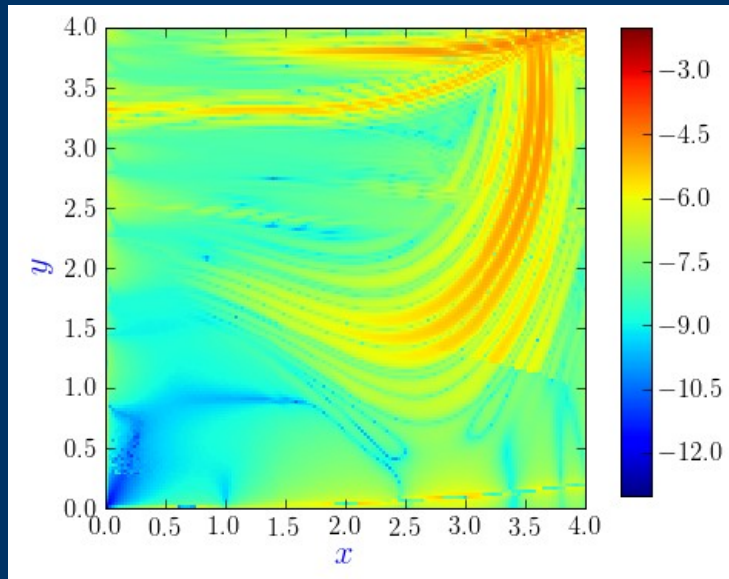


- Initial amplitude particles of 0-4 sigma.
- Resonance line: blue(9<sup>th</sup> order), cyan(12<sup>th</sup> order).
- Wire makes the tune spread wider.
- Resonance line below 12<sup>th</sup> order does not span the footprint.

With Wire

No Wire

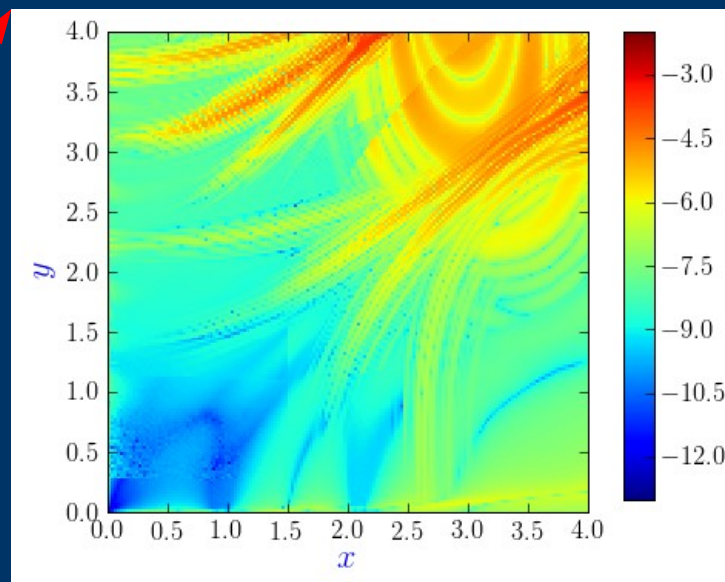
# Gold beam: Freq. Diffusion map (RHIC)



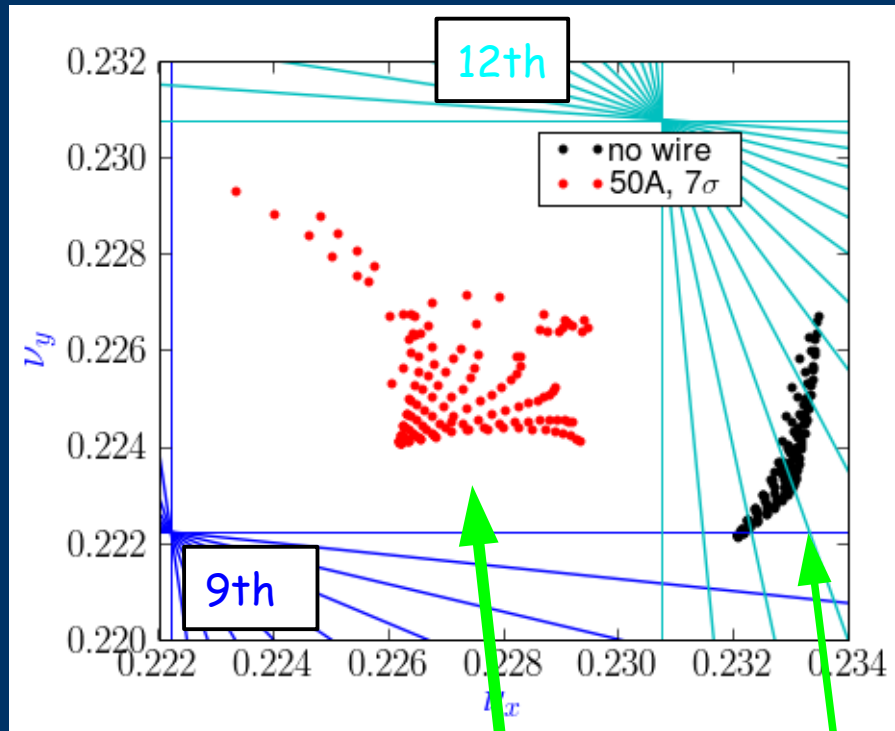
No Wire

Wire:  
50A  
7sigma

- Freq. Diffusion: tune change btwn first and second 1024 turns
  - $DQ = \log[\sqrt{dQx^2 + dQy^2}]$
- Red color corresponds larger diffusion.
- Wire increases the detuning of betatron tune.
- Wire makes the particle motions more chaotic at amplitude beyond 3 sigma.



# Deuteron beam: tune footprint (RHIC)

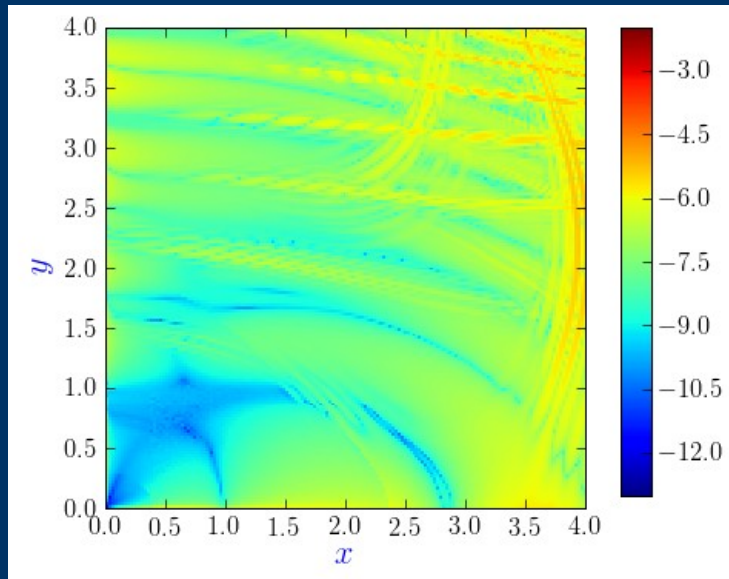


With Wire

No Wire

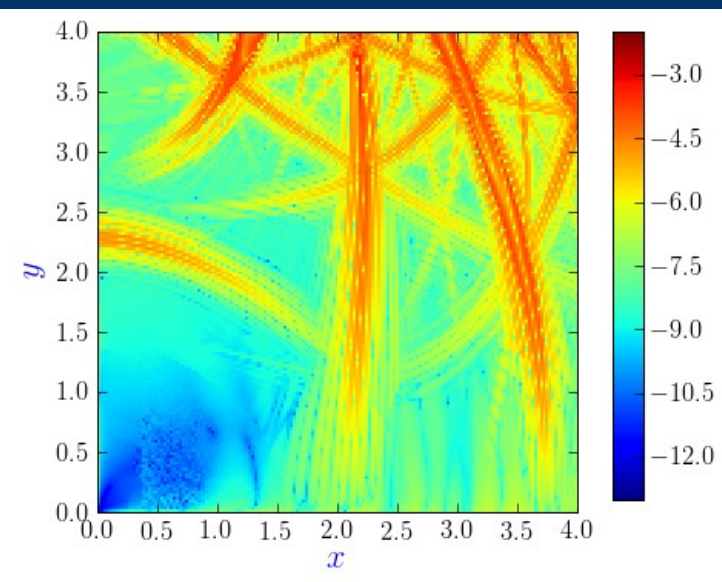
- Initial amplitude particles of 0-4 sigma.
- Resonance line: blue(9<sup>th</sup> order), cyan(12<sup>th</sup> order).
- Final tunes of the deuteron beam due to the wire is closer to the diagonal
- Deuteron beam is free from the 9<sup>th</sup> and 12<sup>th</sup> order resonances.

# Deuteron beam: Freq. Diffusion map (RHIC)



No Wire

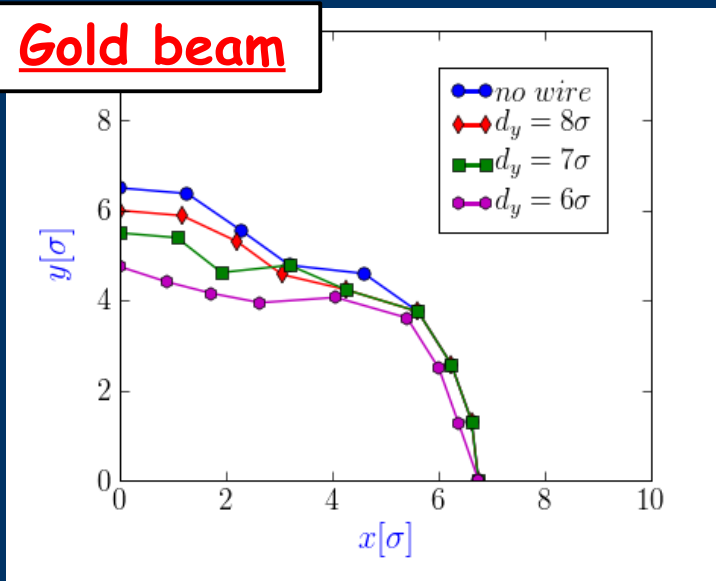
Wire:  
50A  
7sigma



- No wire: mostly stable motion and only a small region with appreciable diffusion (only 12<sup>th</sup> resonance spanning)
- Wire changes the diffusion map significantly.
- Regions with large diffusion are observed even at 1 sigma amplitude even though no resonances below 12th order are spanned by the beam distribution.

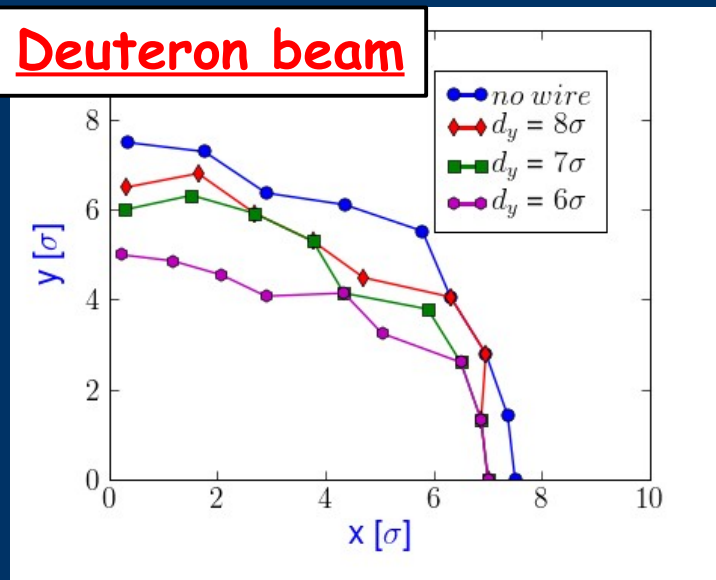
# Dynamic aperture (RHIC)

## Gold beam



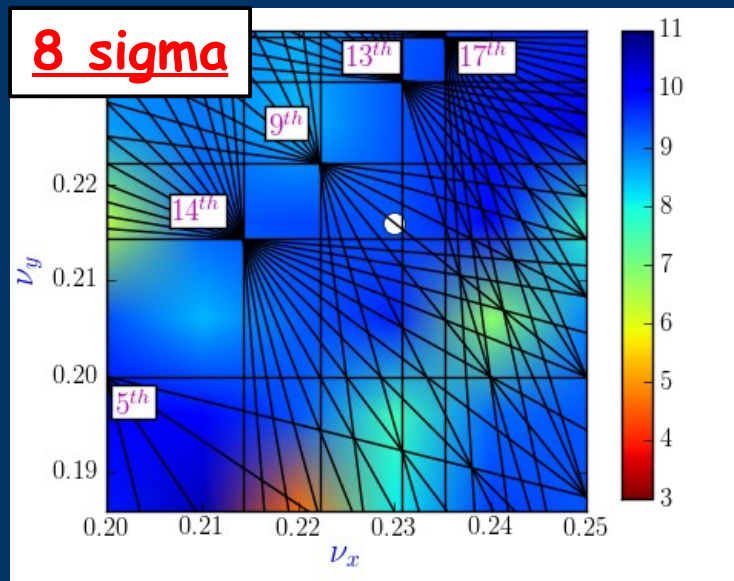
- Dynamic aperture is defined as the largest radial amplitude of particles that survive up to a certain time interval (1E6 turns).
- Wire distorts the boundaries near the vertical plane since the wire is moved in the vertical plane.
- With the wire powered, the DA in the two cases is nearly the same.
- Relative change of DA in Deuteron is bigger than Gold.

## Deuteron beam

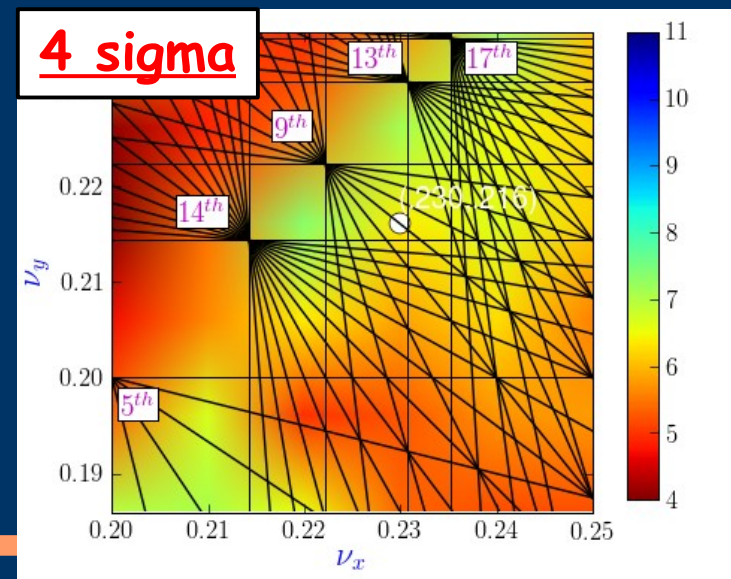
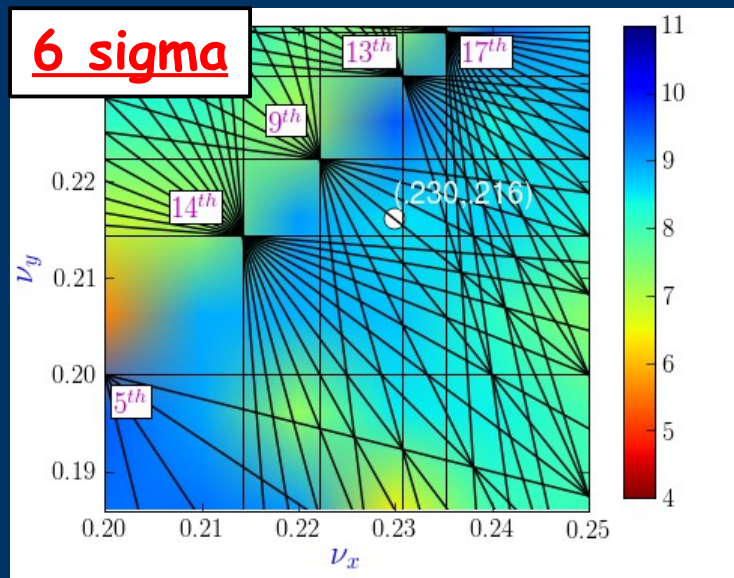




# DA: Tune scan (RHIC gold-gold injection)

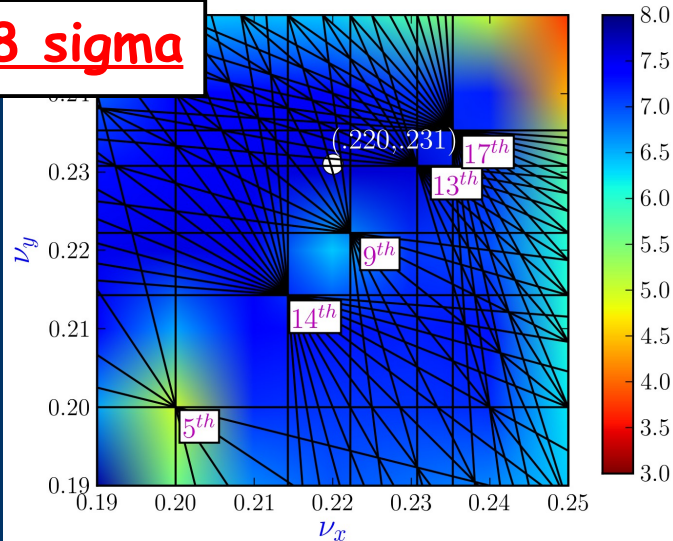


- At all wire separations, the largest dynamic apertures are distributed along the diagonal line  $Q_x - Q_y = 0.02$ .
- The zone along  $Q_x - Q_y = 0.03$  has the smallest dynamic apertures.
- This scan indicates that the nominal tune is close to optimal.
- A sharper drop in dynamic aperture is observed near the 5<sup>th</sup> order resonance.



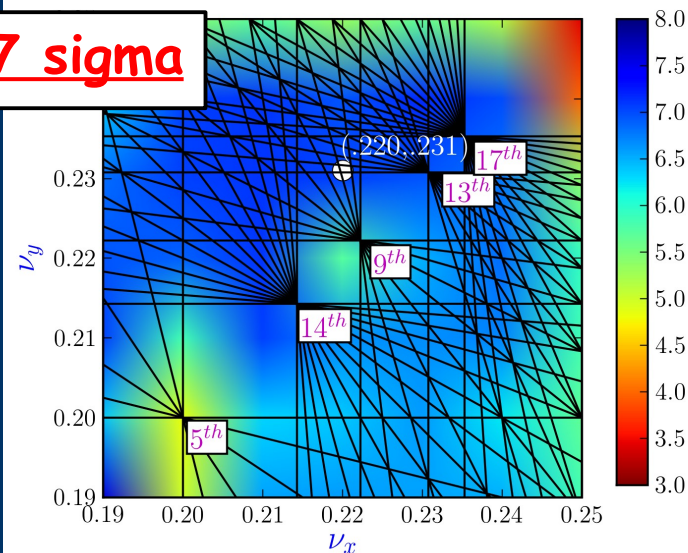
# DA: Tune scan (RHIC gold-gold storage)

## 8 sigma

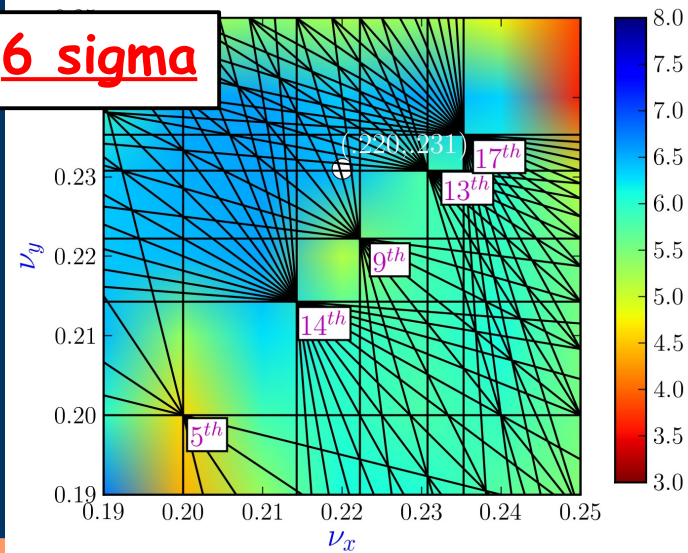


- At all wire separations, the largest dynamic apertures are distributed nearly along the diagonal between  $Q_x=0.21$  and  $Q_x=0.24$ .
- The zone along  $Q_x=0.25$  has the small dynamic apertures.
- Nominal tune is in the region of large DA.

## 7 sigma



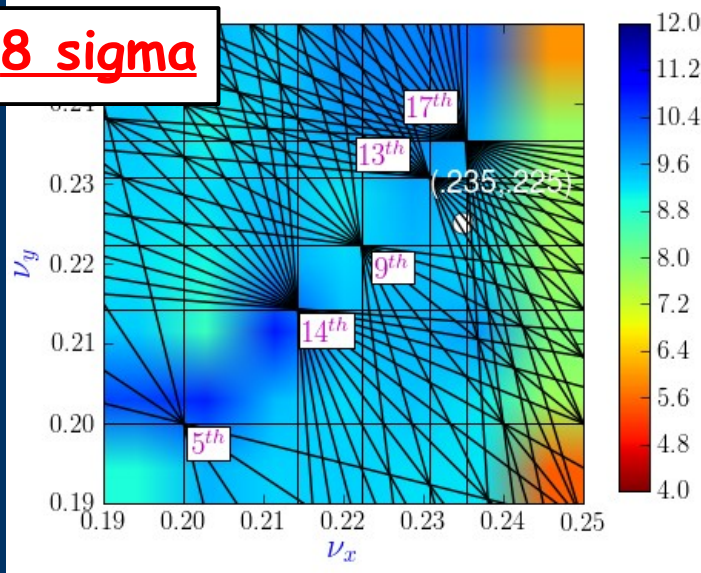
## 6 sigma





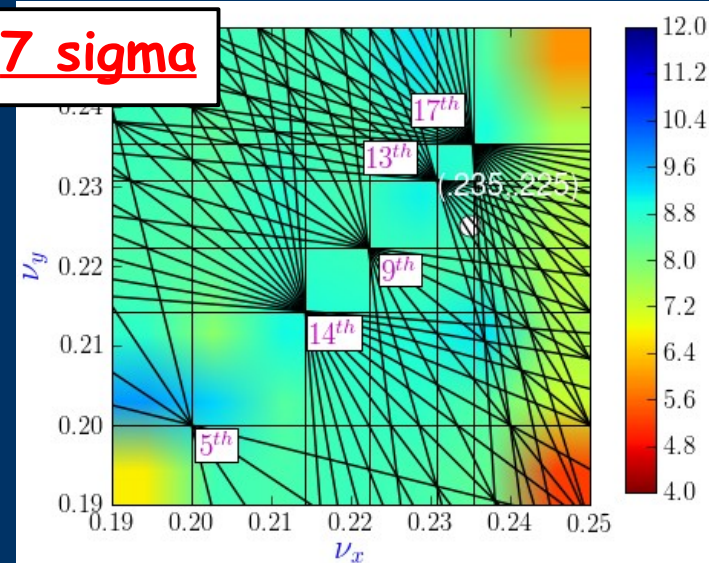
# DA: Tune scan (RHIC deuteron-gold storage)

## 8 sigma

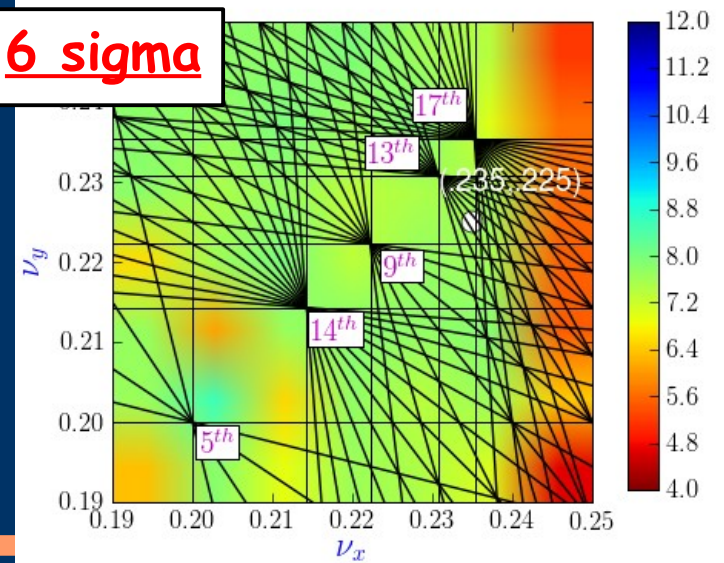


- Reduction of the DA is dominant near 4th resonance.
- A notable variation is seen near a circular band, i.e.,  $Q_x^2 + Q_y^2 = 0.21^2$ , when the beam-wire separation is small.
- Nominal tune is in the region of large DA.

## 7 sigma

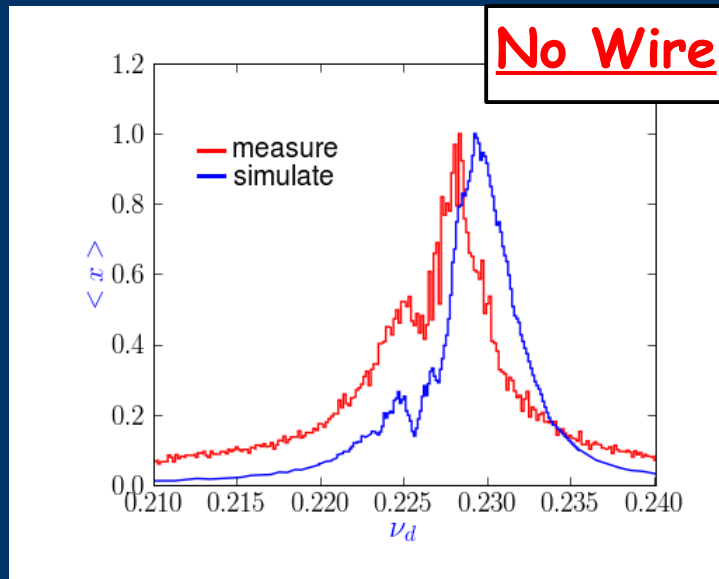


## 6 sigma

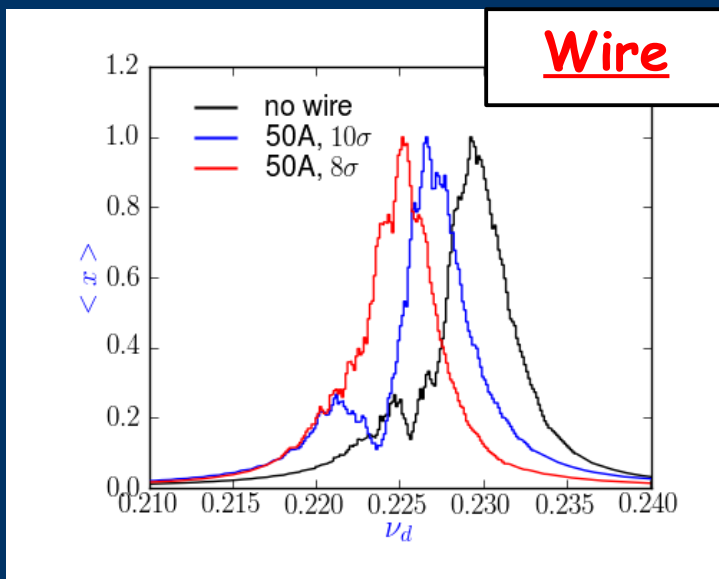




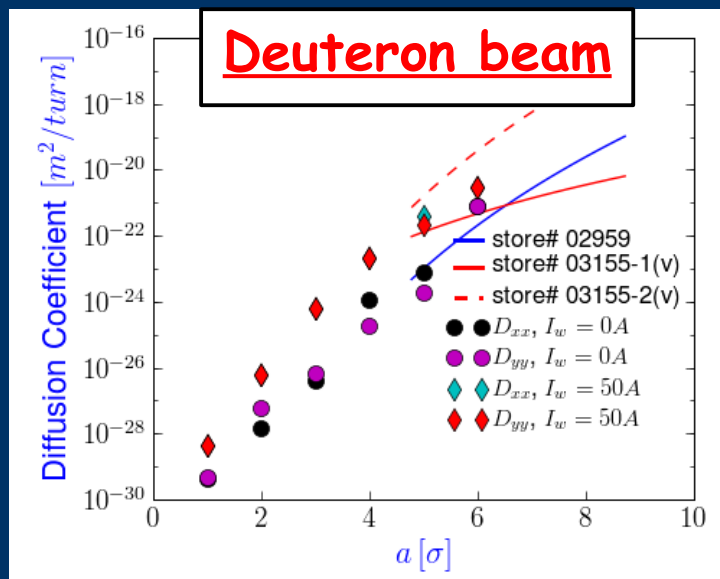
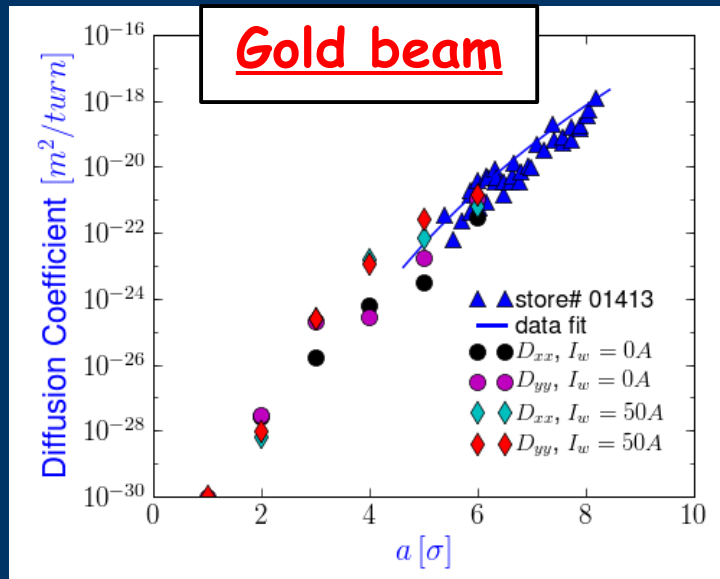
# BTF (beam transfer function) in RHIC



- $\langle x \rangle$ : beam response to a small external transverse excitation at a given frequency.
- Transverse coupling is observed: One peak is close to 0.230 which is the horizontal tune, and the other is 0.225 which is the vertical tune.
- The shift of a peak location of the amplitude increases as the wire separation decreases.
- Width of the amplitude response widens.
- The shift is equivalent to the tune shift of zero amplitude particles.

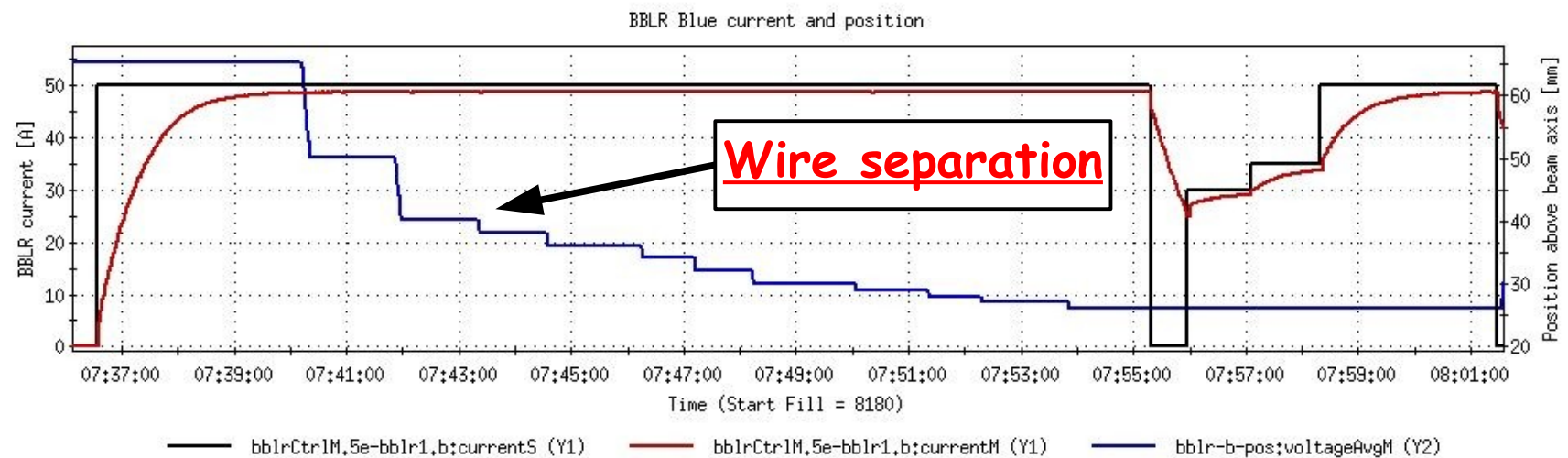
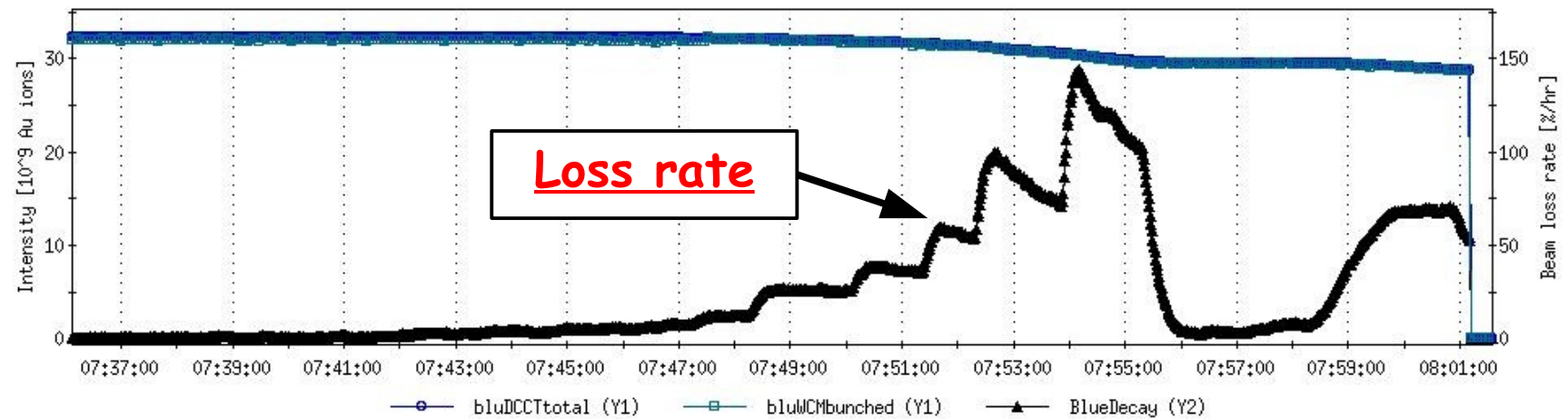


# Particle diffusion (RHIC)

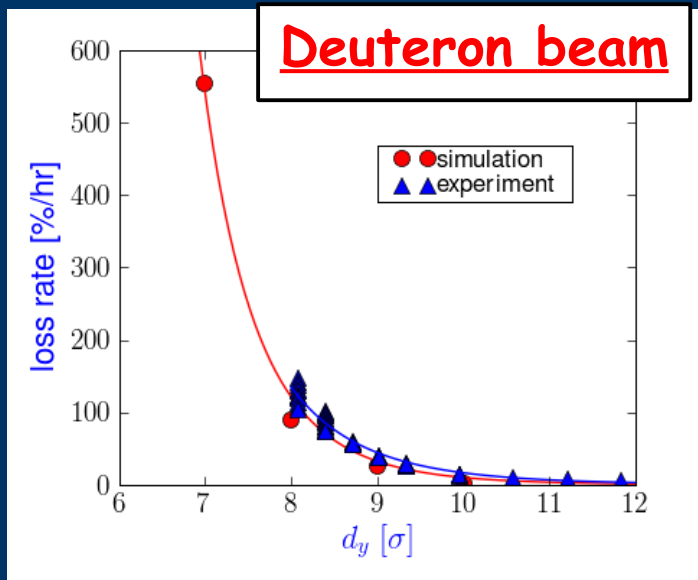
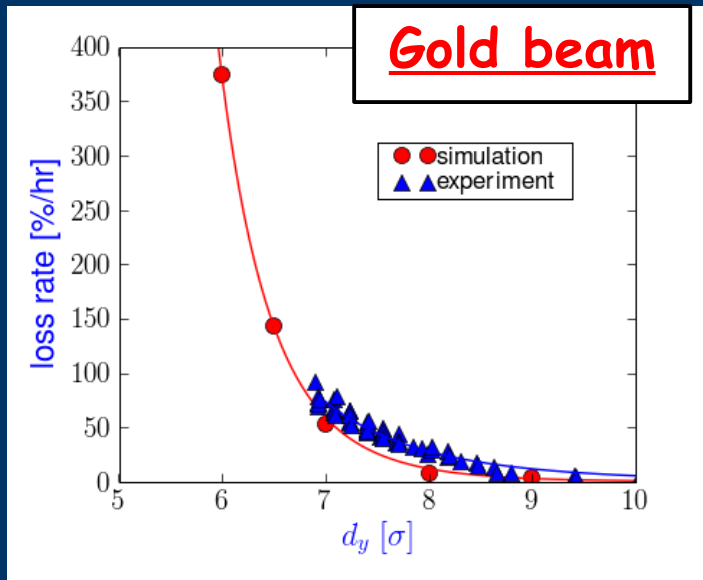


- Simulation:  $D_{xx}(a) = \frac{1}{N} \left\langle (J_x(a, N) - J_x(a, 0))^2 \right\rangle$
- Measurement: obtained by fitting the time-dependent loss rate after moving a collimator into and out from the beam.
- Dependence of diffusion coefficients on the initial action is exponential at small amplitudes and power law-like at larger amplitudes.
- Relative increase of diffusion coefficients at below 3 sigma amplitude for the deuteron beam is higher than that for the gold beam.
- Enhanced diffusion at near 3 sigma amplitude for the deuteron beam leads to significant increase of particle loss under the simulation conditions.

# Loss rate due to beam-wire interaction (2008)



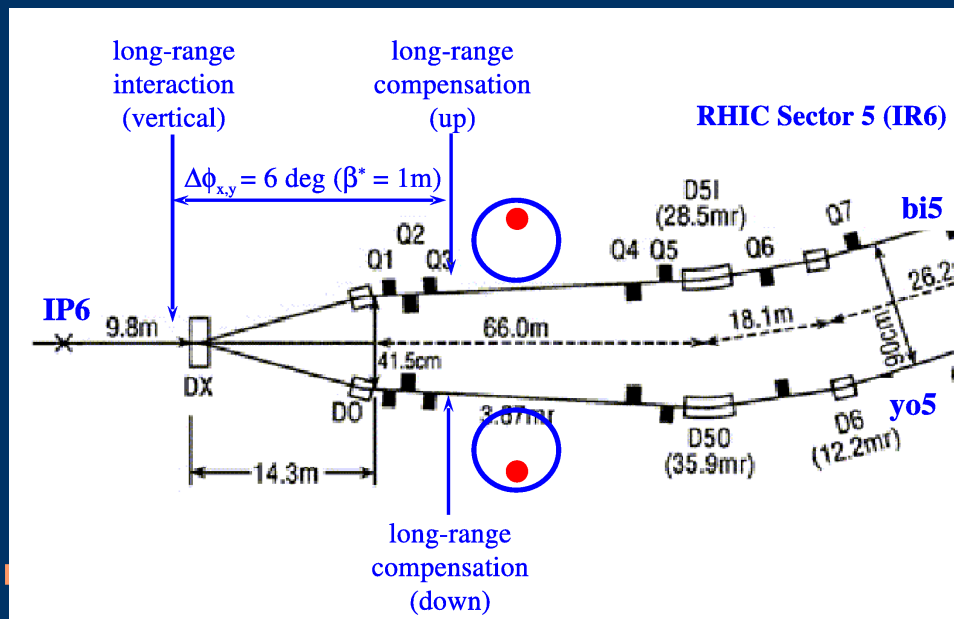
# Particle loss rate (RHIC)



- Onset of beam losses is observed at 8 and 9 sigma for gold and deuteron beams.
- Separation at which there is a sharp rise in the loss rates agree with measurement.
- At fixed separation, loss of deuteron beam is higher than gold beam.
- Freq. diffusion with the wire shows greater diffusion in the deuteron case.
- Action diffusion is also larger in the deuteron beam.
- Both frequency and action diffusions seem to be better correlated with loss rates than the traditional short term indicators like footprints and dynamic aperture.

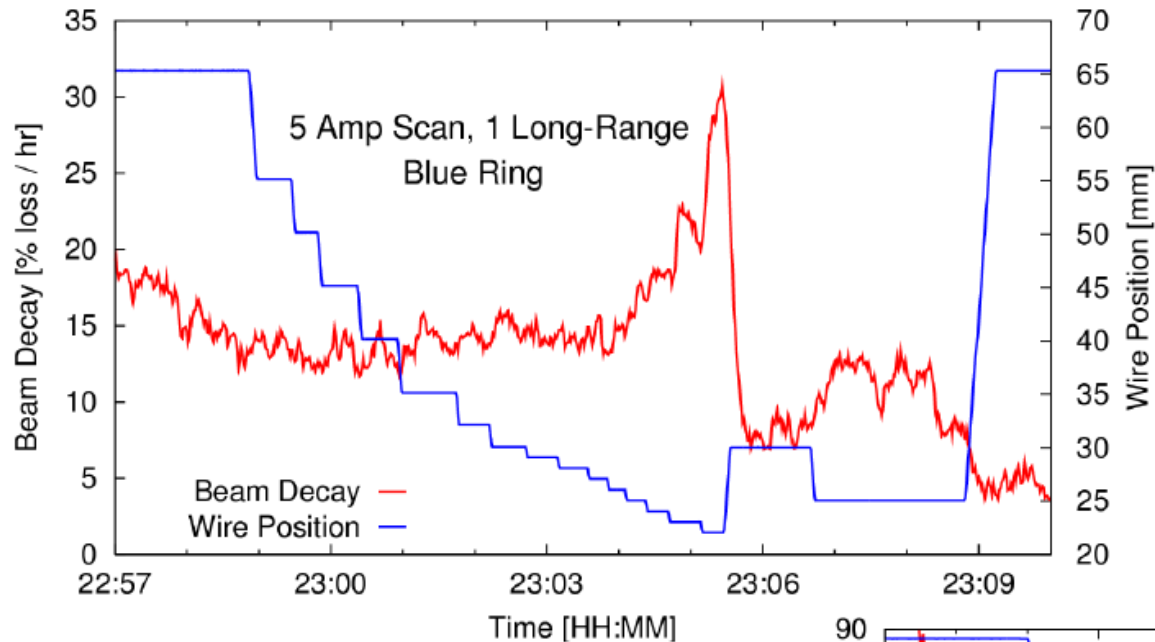
# Long-range compensation (May 27, 2009)

- 100Gev proton-proton beam
- Bunch intensity:  $1.7E11$  p/bunch
- Yellow: tune(0.695,0.692), chrom(-1.5,1.0),  $\epsilon(49,19)$
- Blue: tune(0.691,0.688), chrom(2.3,-1.4),  $\epsilon(24,-)$
- Single long-range interaction near DX magnet.
- Wire current: 5A

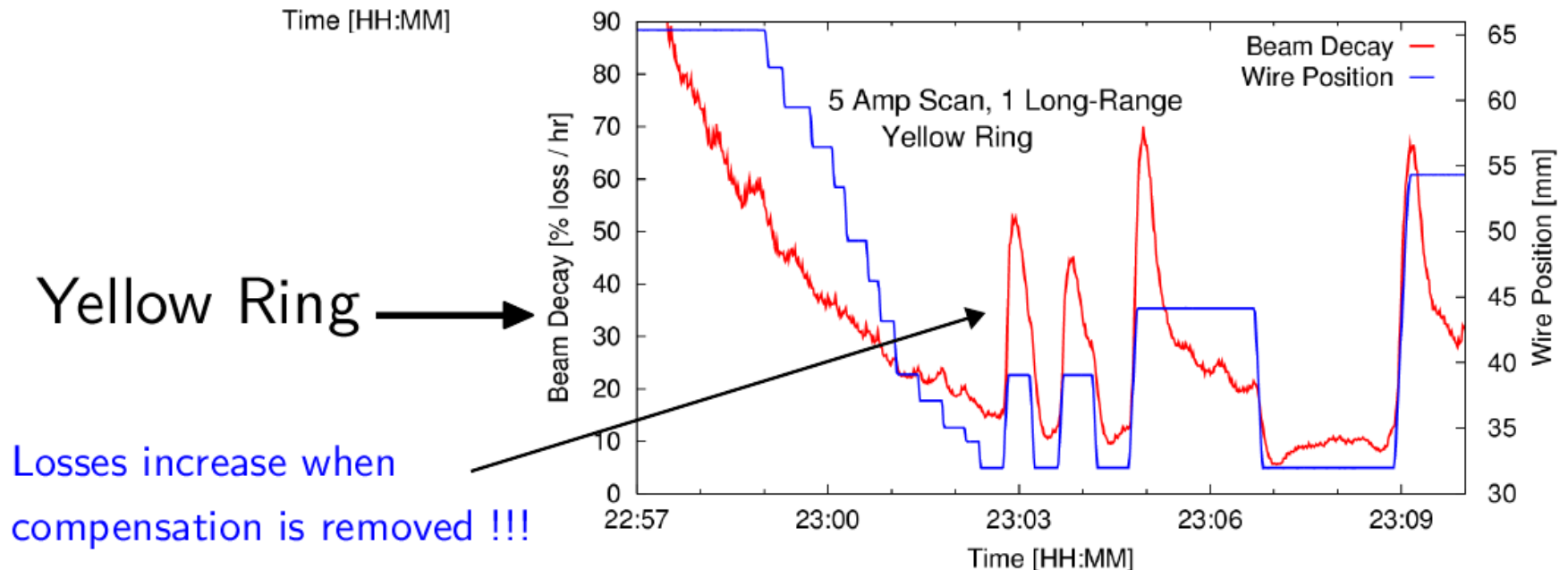


R. Calaga, W. Fischer, G. Robert-Demoliaze

# Long Range "Compensation" - Yellow



Blue Ring  
No visible effect



## *Summary (beam-wire interaction in RHIC)*

- Betatron tune change due to the wire is well tracked by the simulation.
  - Wire causes a significant increase in tune spread and diffusion for both gold and deuteron beams.
  - Stability boundary near the vertical axis is linearly proportional to the beam-wire separation.
  - Tune scan of DA identifies the betatron tune where DA is maximized for both gold and deuteron beams.
  - BTF simulation and measurement identify betatron tune and transverse coupling.
  - Action diffusion for the deuteron beam is larger than for the gold beam.
  - Threshold separation at which there is a sharp rise in the loss rates agree to better than 1 sigma.
  - Tune and action diffusions are closely related to particle loss rate.
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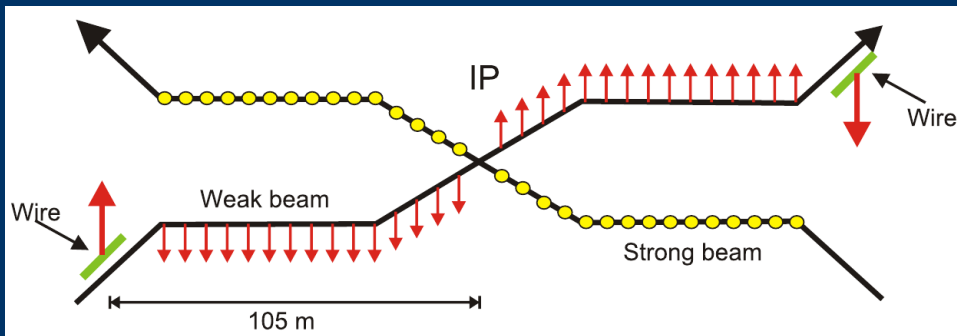
**Beam-beam compensation with current  
carrying wire at LHC**



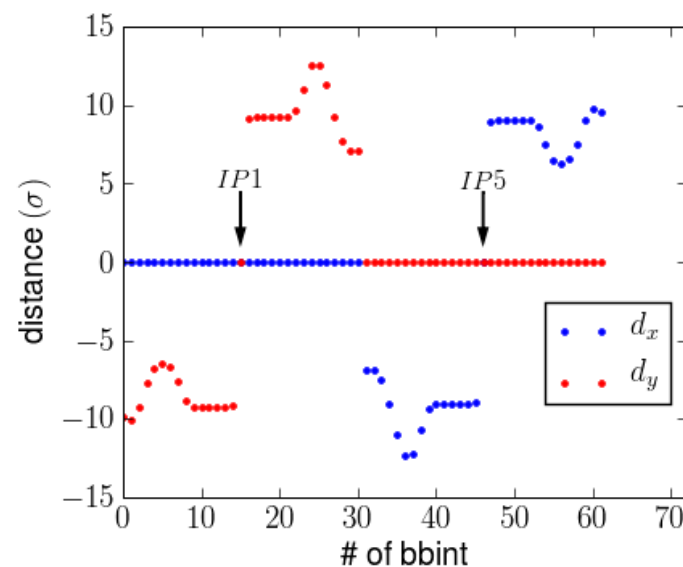


# MODEL: Wire compensation in LHC

- Nominal LHC: 2808 bunches → 30 parasitic crossings per IP
- A wire on each side of IP (total 4)
- Wire strength:  $(IL) = 83 \text{ A-m}$
- Wire location: 105 m for IP
- At wire location:  
 $(\text{betax}, \text{betay}) = (1783, 1792)$

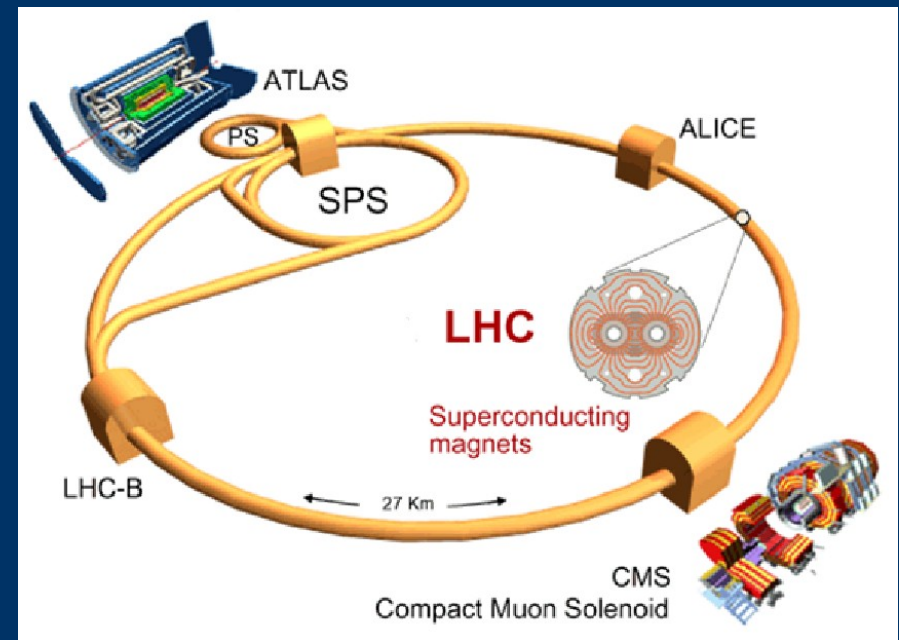


wire	Wire separation (sigma)	
	horizontal	vertical
IP1_left	0	-8.56
IP1_right	0	+9.56
IP5_left	-9.33	0
IP5_right	+8.33	0

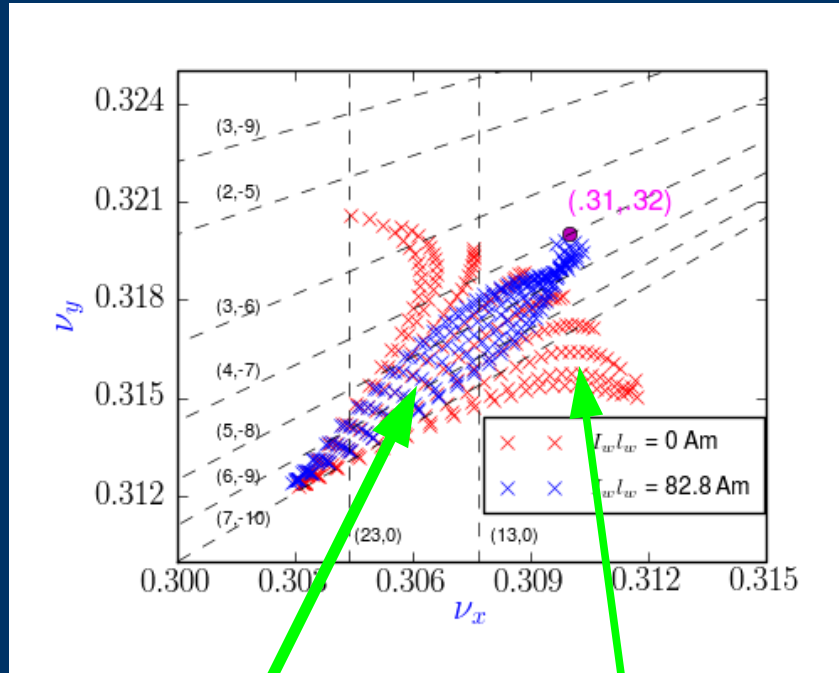


# MODEL: Wire compensation in LHC

- 7 Tev proton-proton beam
- 2 head-on (IP1 & 5),  $\beta^* = 0.55\text{m}$
- Beam intensity:  $1.15\text{E}11$  per bunch
- Crossing angle: 285 micro-rad
- Working point: (0.31,0.32)
- Chromaticity: (+2,+2)
- Emittance: 22.5 mm-mrad



# Footprint (LHC)

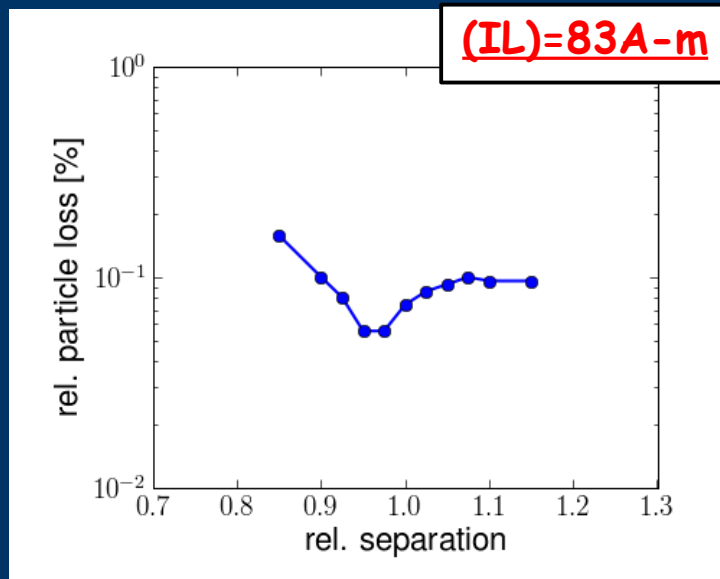
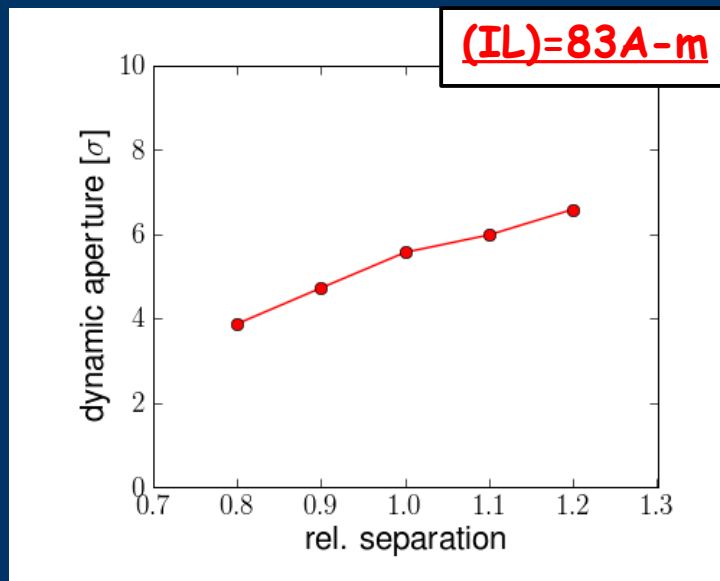


With Wire

No Wire

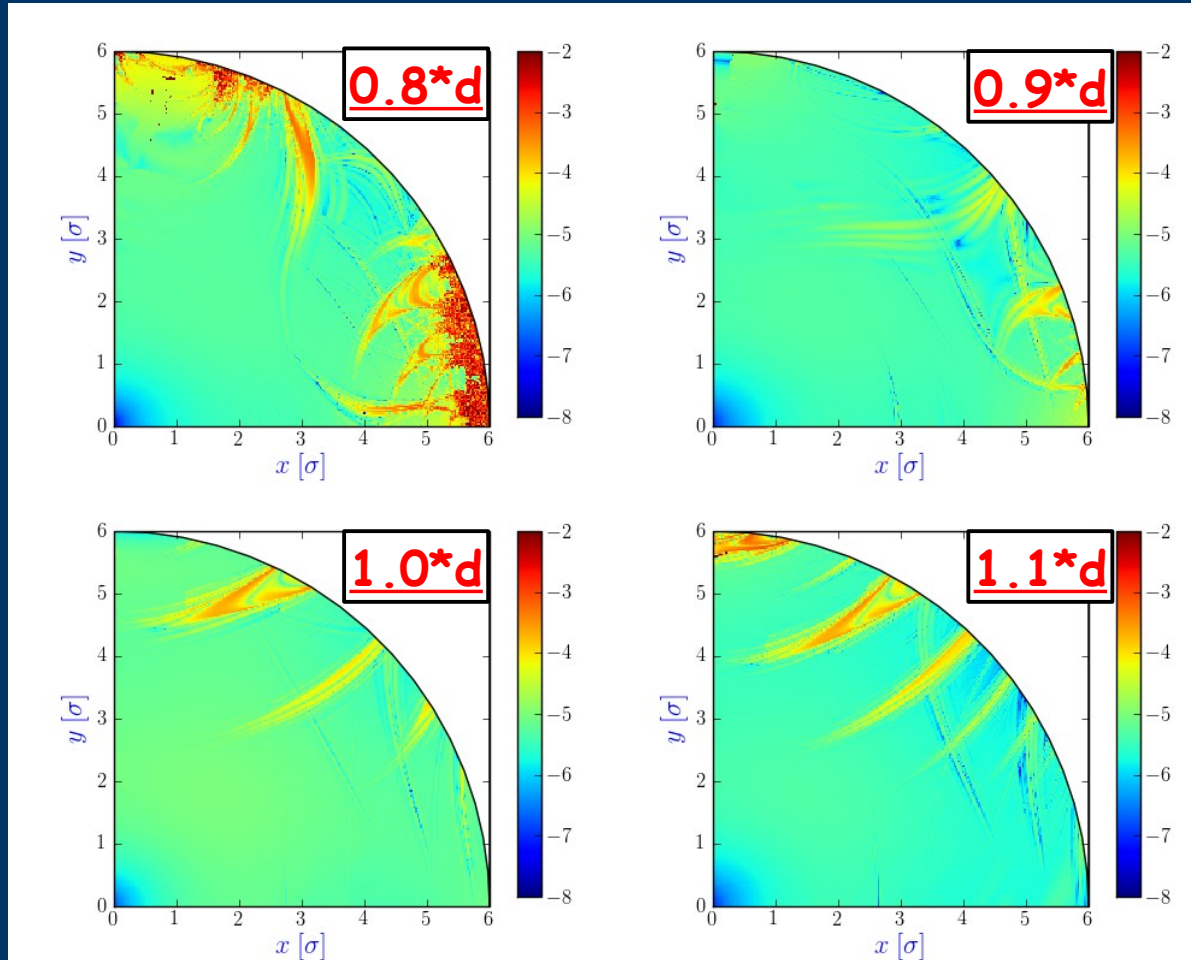
- beam-beam parameter is 0.004.
- long-range interaction affects higher amplitude particles.
- Long-range interaction increases the tune spread of the high amplitude particles.
- footprint can be compressed to nearly the same spread as with the long-range interactions excluded.

# Wire position scan: DA / beam loss (LHC)



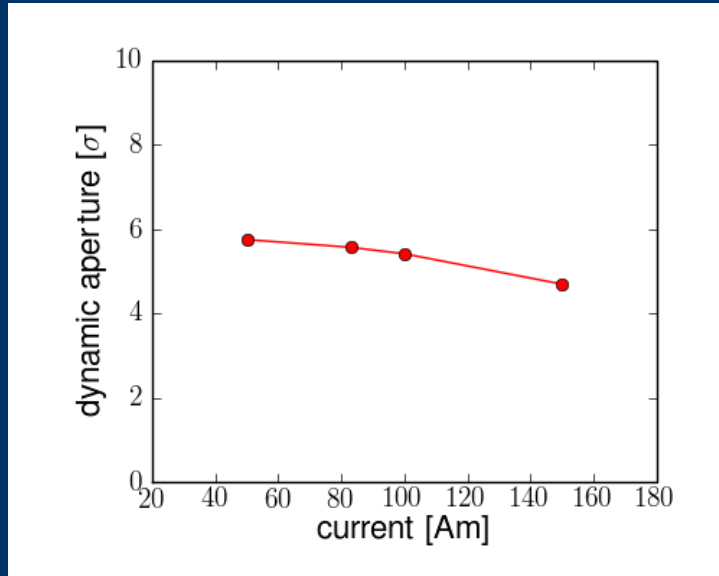
- wire-beam separation distance is one of major wire parameters.
- separation is relative to average beam separation.
- angle-averaged dynamic aperture for off-momentum particles with  $dp = 3 \text{ sigma}$ .
- dynamic aperture decrease linearly as the separation decreases.
- minimum particle loss between 0.9 and 1.0 separations.
- Proposed separation is close to optimal one.

# Freq. diffusion vs. wire separation (LHC)



- Small amplitude particles are unaffected by the beam-beam compensation.
- Freq. diffusion is improved at a certain separation (0.9 and 1.0 separations).
- suppress the tune change at large amplitude beyond 4 sigma.

## DA vs. current (LHC)



- Current is varied from 40 Am to 150 Am ( 0.5 - 2 times 82.8 Am).
- DA stays roughly constant up to 100 Am, and falls down to 4.5 sigma.
- Beam dynamics are less sensitive to wire current than wire-beam separation.

		DA
separation	0.8-1.2	4 - 7 sigma
current	0.5-2.0	4.5-5.5 sigma

## *Summary (wire compensation in LHC)*

- The results show that the particle loss is minimized at the wire separation between 0.9 and 1.0 of the reference separation.
- 
- The separation corresponds to the one where the tune change of large amplitude particles is reduced.
- 
- The dynamic aperture results show that the beam dynamics are more sensitive to the wire-beam separation than the wire current.

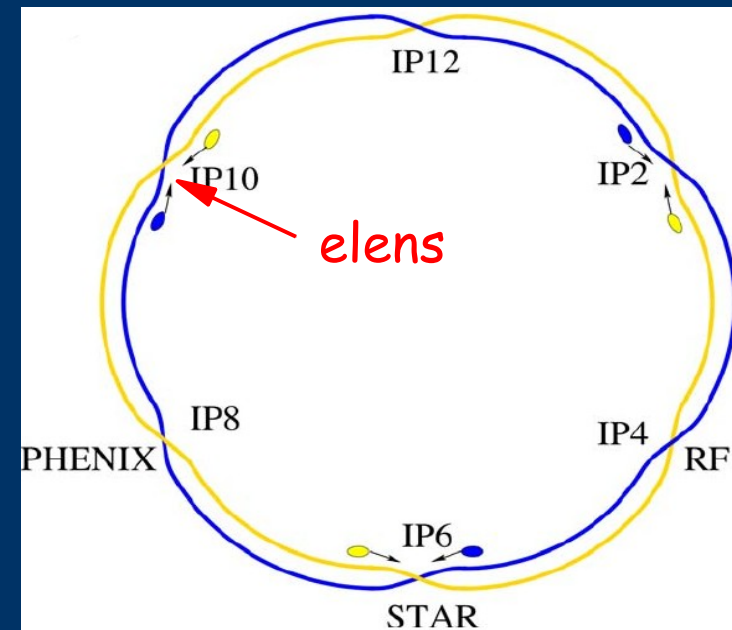
# *Beam-beam compensation with electron lens at RHIC*





## MODEL: Electron lens simulation at RHIC

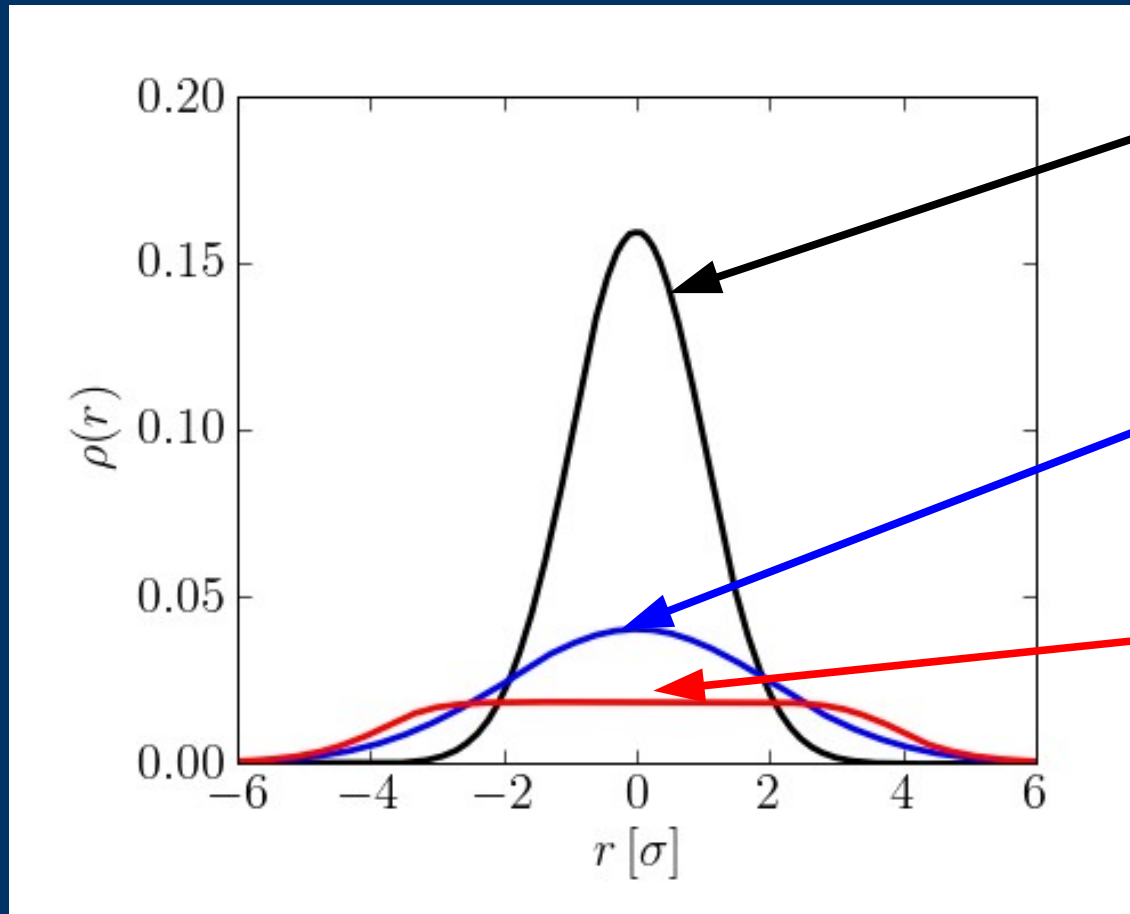
- 1.7E11 bunch intensity is achieved at Run-08.
  - For  $> 2E11$  intensity, large beam loss is expected (2/3, 7/10 resonance).
  - Elens installation by end of 2011.
- 
- 250 GeV p-p beam
  - 2 head-on (IP6 & 8),  $\beta^* = 0.5\text{m}$
  - Beam intensity: 2E11 per bunch
  - Working point: (0.695, 0.685)
  - 1 e-lens at IP10,  $\beta = 10\text{m}$
  - NL: sextupoles/IR multipoles



## Electron Lens Requirement

- **For full tune-spread compression**
    - Electron beam profile should match proton profile at IP ( Gaussian)
    - Electron beam intensity should be  $N_e = N_{ip} * N_p$  ;  $N_{ip} = 2$ ,  $N_p = 2E11$
    - Full tune-spread compression does not help to reduce particle loss (BBSIMC, LIFETRAC, SIXTRACK)
  - **For reduction of particle loss**
    - Electron beam profile should match proton profile for tune compression, but other profiles may be more suitable for reducing particle loss.
    - Electron beam intensity may be different from  $N_{ip} * N_p$
- 
-

# Electron beam profiles



1 sigma Gaussian

-  $\exp(-0.5(r/\sigma)^2)$   
- match to proton beam size

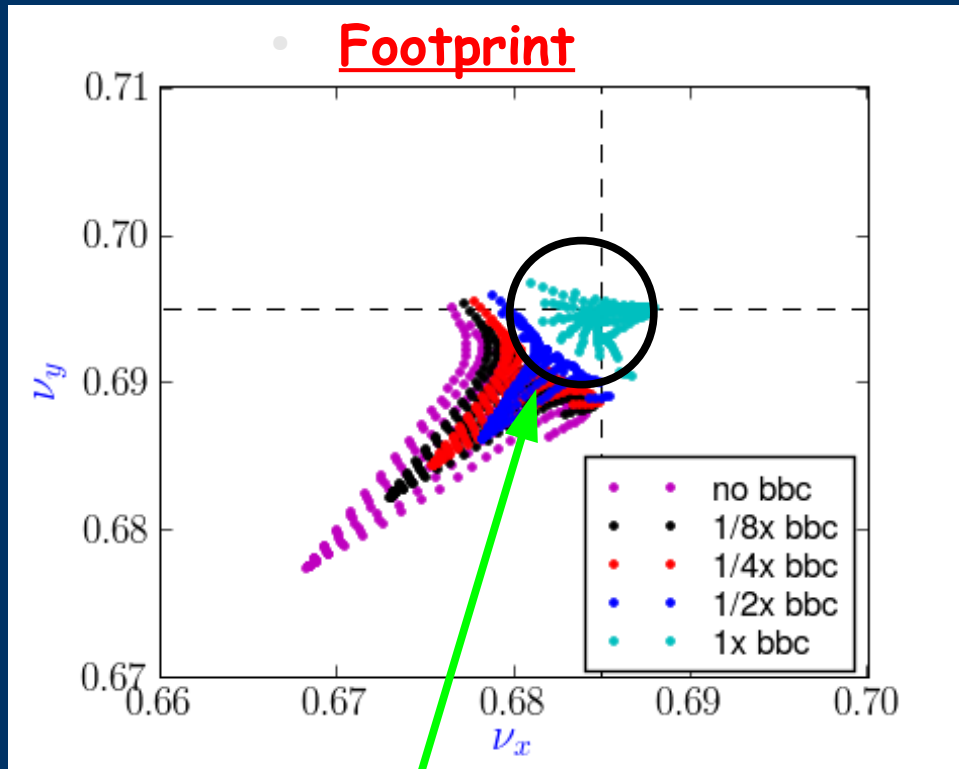
2 sigma Gaussian

-  $\exp(-0.5(r/2\sigma)^2)$

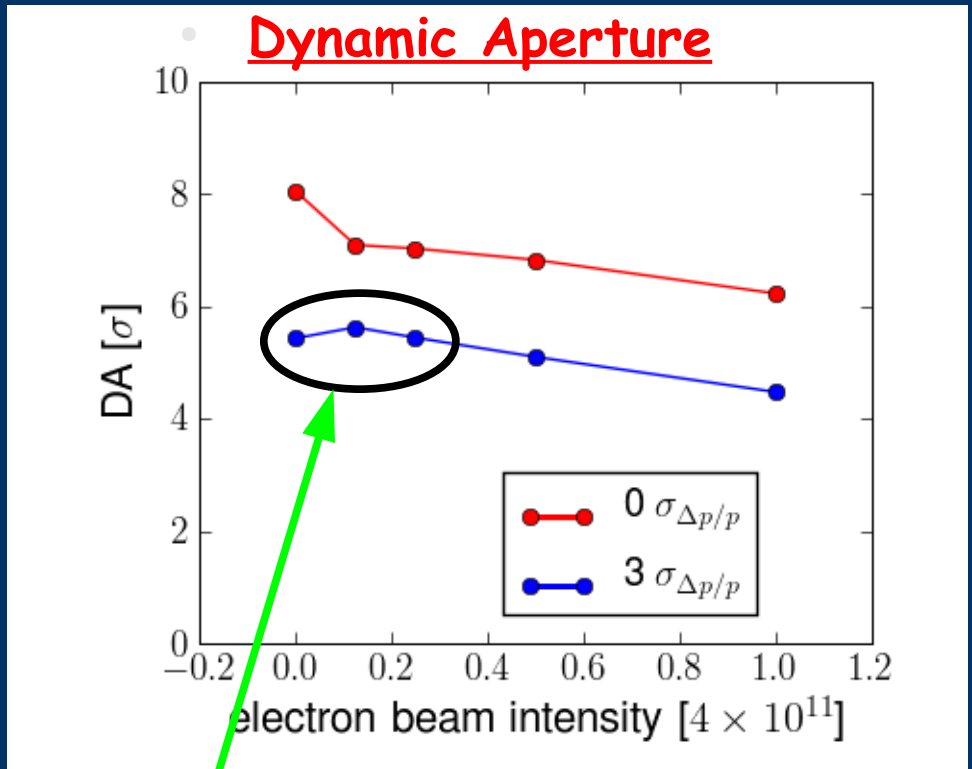
Smooth Edge Flattop(SEFT)

-  $1/(1+(r/4\sigma)^8)$

## Gaussian Electron Lens ( 1 sigma)



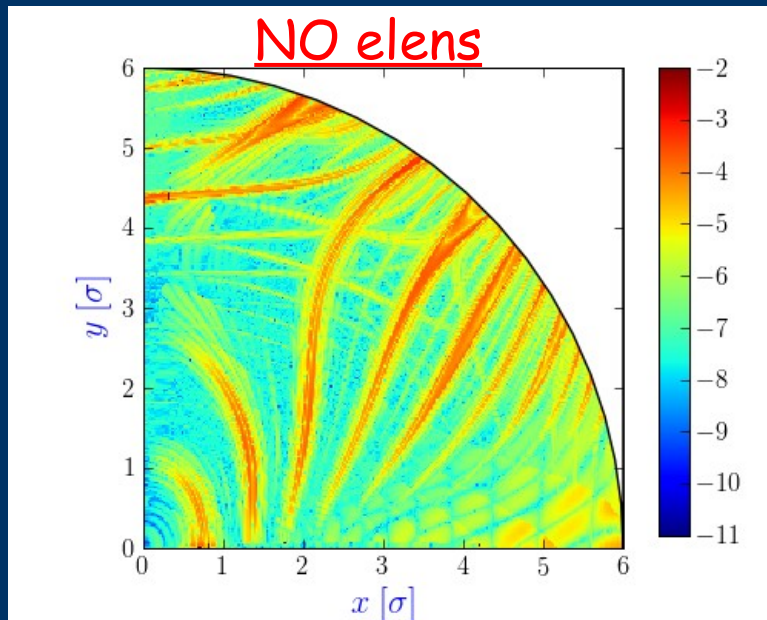
1x bbc fully compensates footprint. Footprint folding is observed.



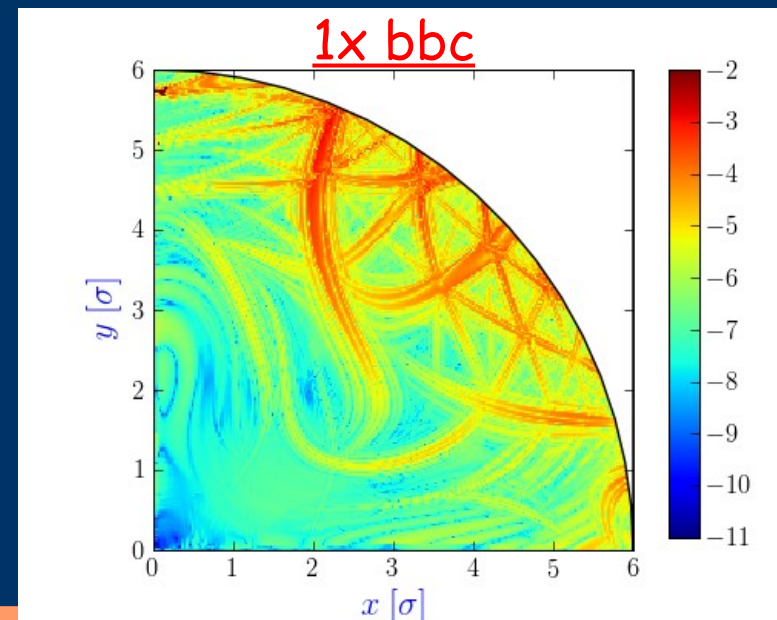
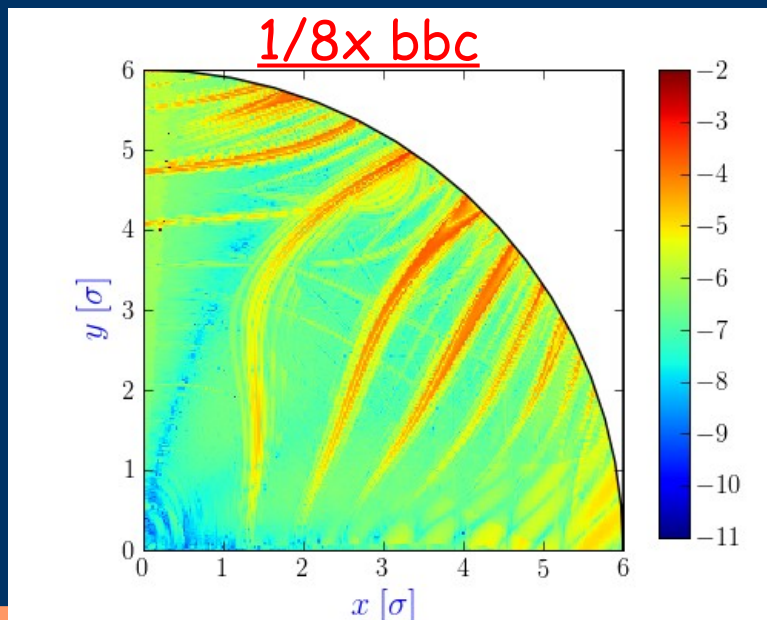
DA is increased at 1/8x bbc

- 1x bbc = beam-beam compensation with  $N_e = N_{ip} * N_p = 2 * 2E11$

## Gaussian Electron Lens ( 1 sigma)

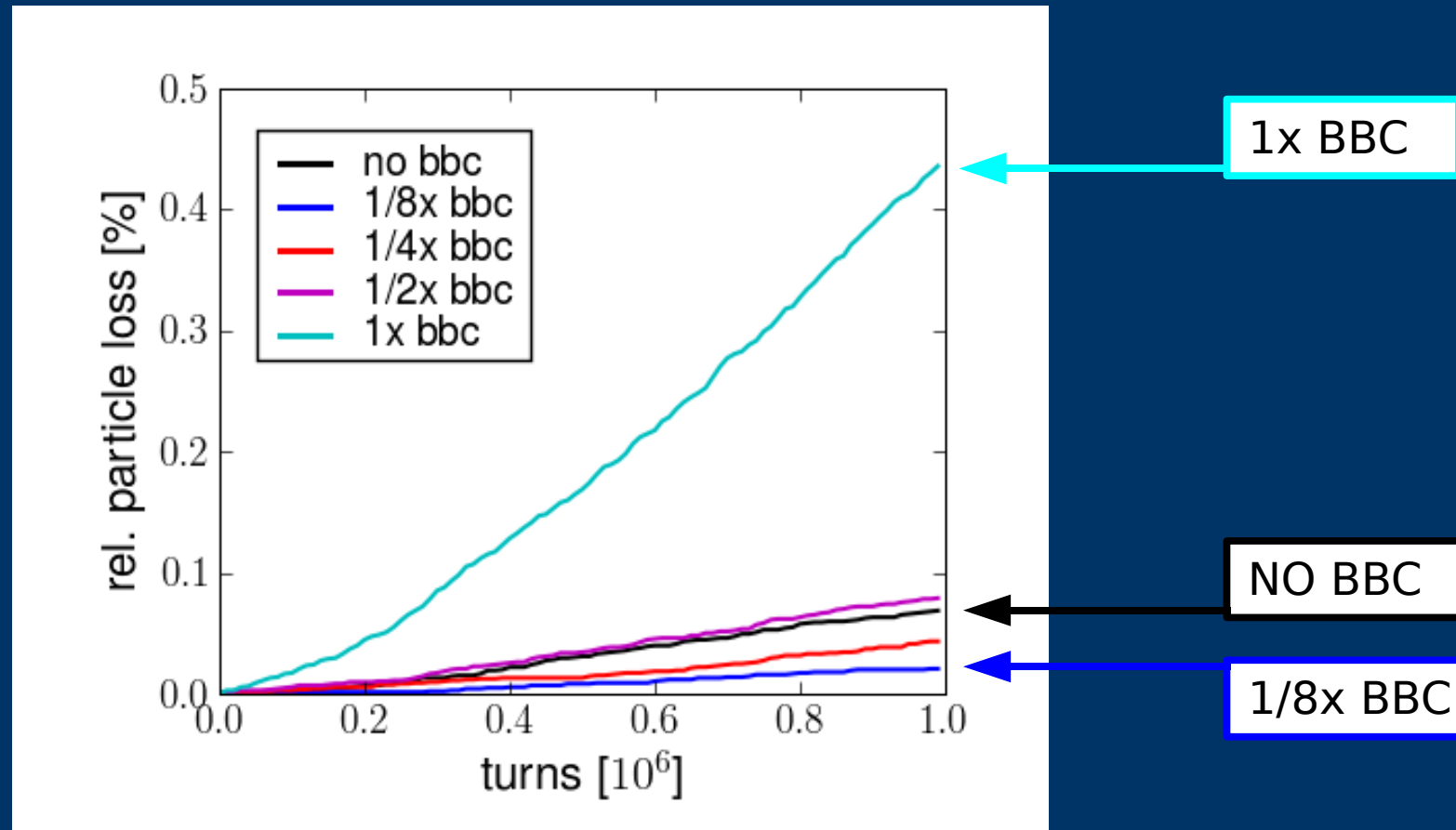


- Freq. diffusion: tune change btwn first and second 1024 turns
  - $DQ = \log[\sqrt{dQx^2 + dQy^2}]$
- 1x bbc: decrease tune change at small amp. but increase at large amp.
- 1/8x bbc: decrease tune change at both small and large amp.



## Gaussian Electron Lens (1 sigma)

- Particle loss



- Small  $N_e$  reduces beam loss:  $N_e < 0.5 N_{ip} * N_p$ 
  - (loss with 1x bbc)/(loss with NO bbc) ~ 600%
  - (loss with 1/8x bbc)/(loss with NO bbc) ~ 30%

## Comparison of electron beam distributions

Profile	Intensity ( $N_{ip} \cdot N_p$ )	Dynamic aperture (sigma)	Particle loss (Relative to NO elens)
1 sigma Gaussian	1/2	5.10	115%
	1/4	5.44	63%
	1/8	5.63	30%
2 sigma Gaussian	2	5.05	10%
	1	5.40	8%
	1/2	5.63	6%
SEFT	2	4.77	22%
	1	5.47	6%
	1/2	5.57	6%

- Below threshold current with 2 sigma Gaussian and SEFT profiles, particle loss is relatively insensitive to electron lens current variations.

## Summary (elens compensation in RHIC)

- Full tune-spread compression causes footprint folding and increases particle loss. Partial tune-spread compression without inducing footprint folding may reduce particle loss.
- Tune diffusion is closely related to particle loss.
- There is a threshold electron beam intensity below which beam life time is increased

Profile	Threshold ( $N_{ip} \cdot N_p$ )
1 sigma G	0.5
2 sigma G	2
SEFT	4

- Particle losses for 2 sigma Gaussian and SEFT profiles are relatively insensitive to intensities below threshold.
- Wider electron beam profile than proton at elens location is found to increase beam life time. Validation with better statistics in progress.



## Summary

- Simulations of wire-beam interaction in RHIC agree well with experiments.
  - Measurements with wire compensation in RHIC are in progress.
  - Wire compensation in LHC reduces beam loss and the proposed wire separation distance is close to optimal.
  - Electron lens is beneficial to reduction of beam loss in RHIC. Wider electron lens profiles are better.
- 
-

*Thank you*

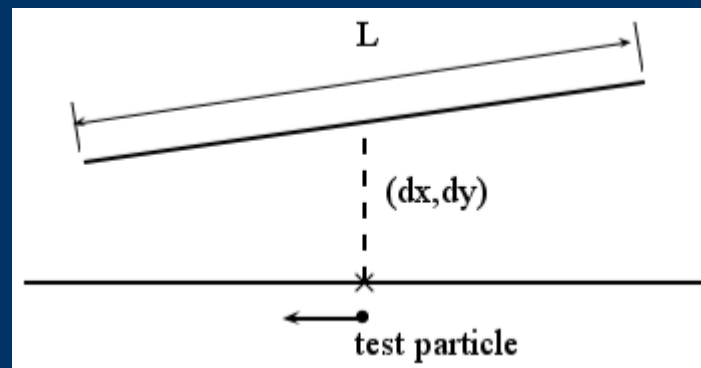


## Electromagnetic lens (current carrying wire)

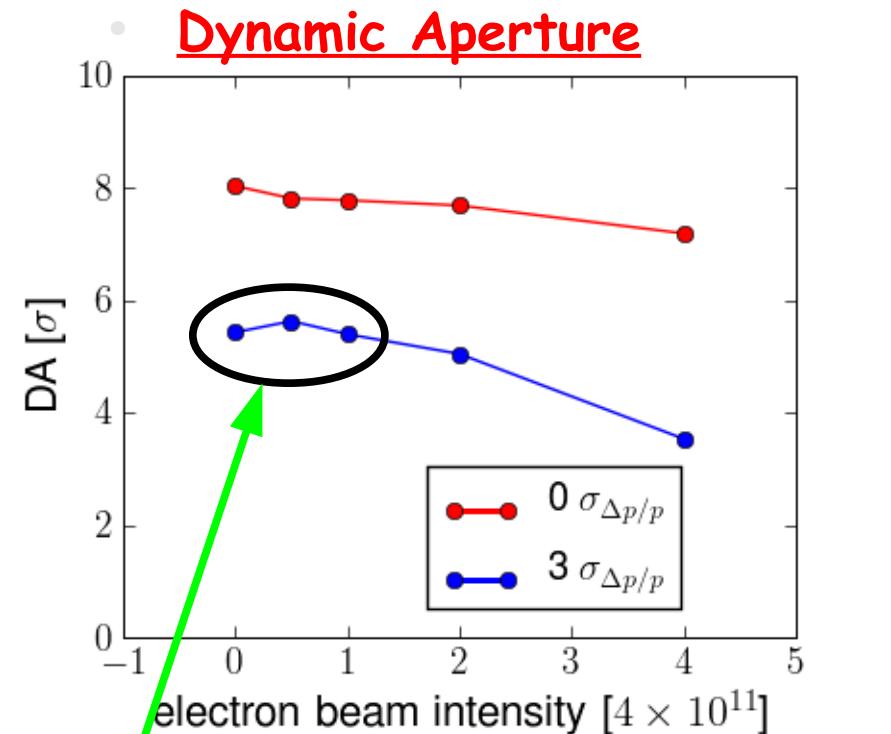
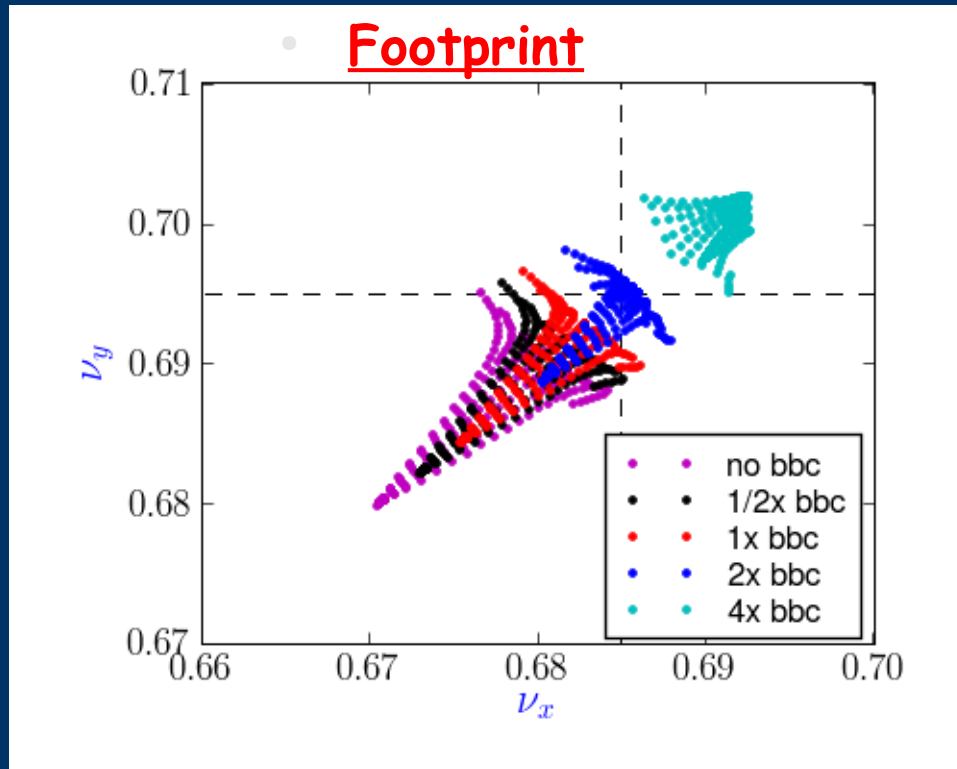
- For a finite length of a wire embedded in the middle of a drift and tilted in pitch and yaw angles, the transfer map of a wire is

$$\mathcal{M}_w = T_{\theta_x, \theta_y}^{-1} \odot D_{-L/2} \odot \mathcal{M}_k \odot D_{-L/2} \odot T_{\theta_x, \theta_y}$$

, where T represents the tilt of the coordinate system by horizontal and vertical angles to orient the coordinate system parallel to the wire, D is the drift map with a length L/2, and M is the wire kick integrated over a drift length



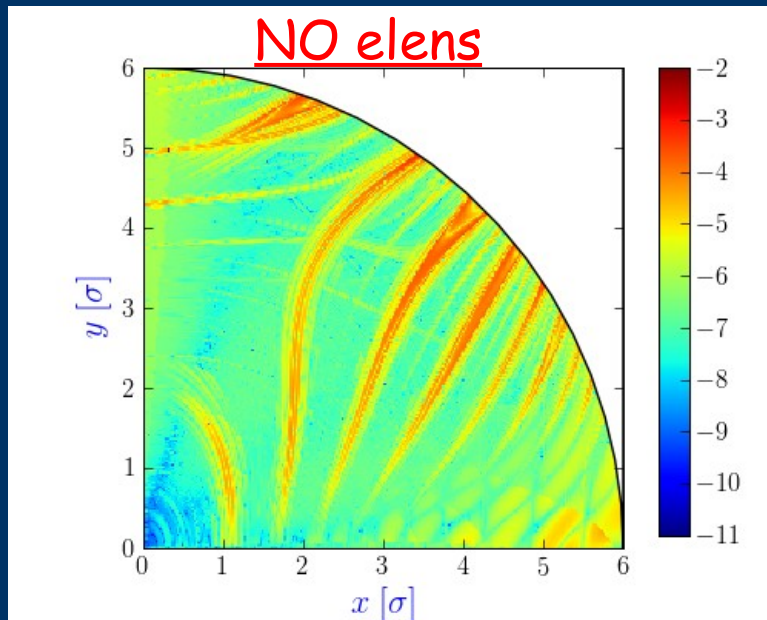
## Gaussian Electron Lens ( 2 sigma)



DA is increased at 1/2x bbc

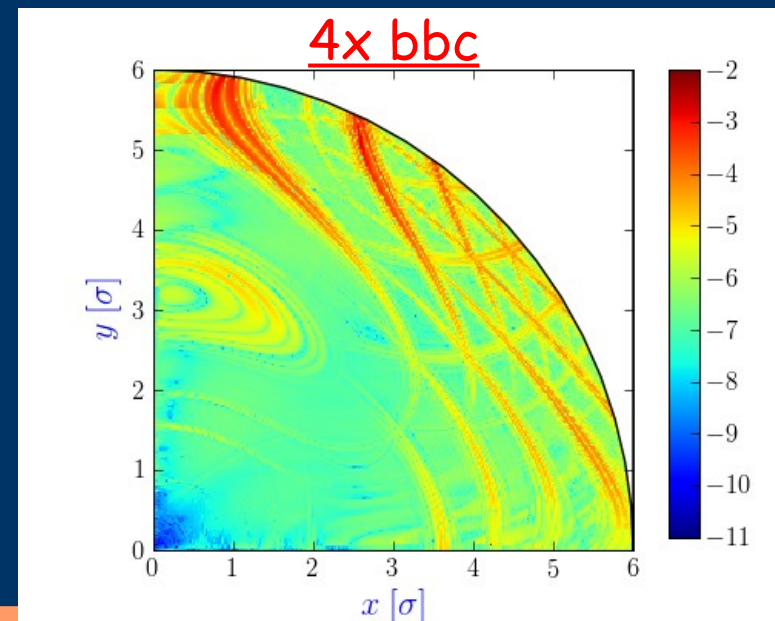
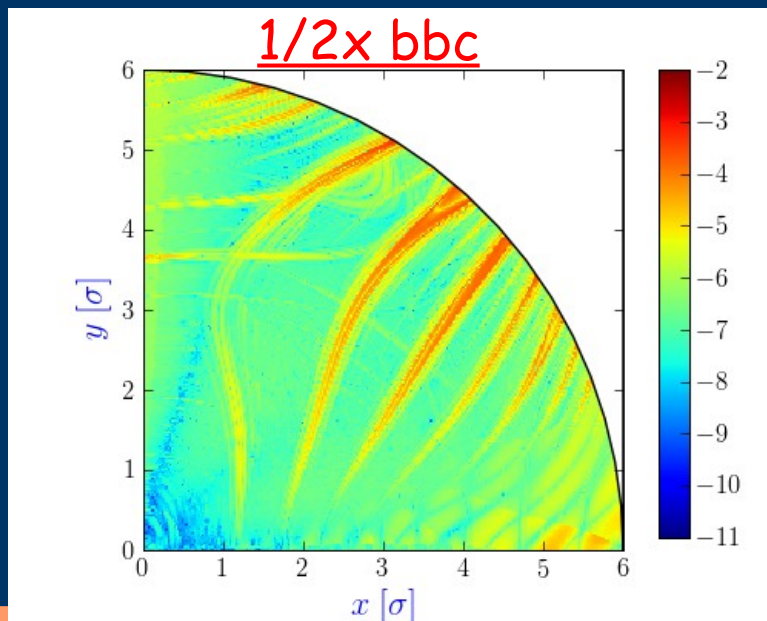
- Peak of 4x bbc electron beam profile is matched to that of 1x bbc at 1 sigma Gaussian.

## Gaussian Electron Lens ( 2 sigma)



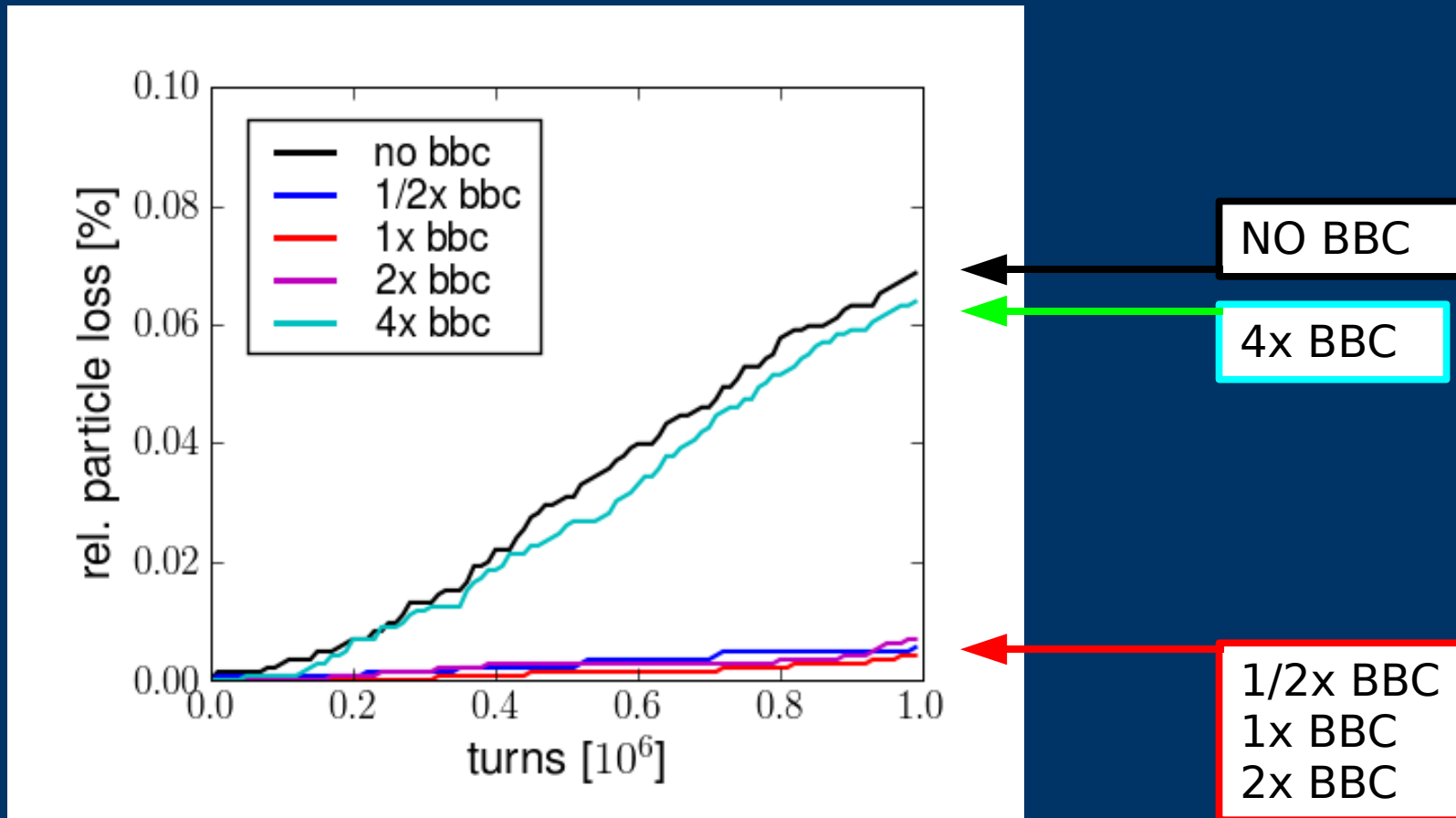
- Tune Diffusion

- 4x bbc: decrease tune change at small amp. but increase at large amp.
- 1/2x bbc: decrease tune change at both small and large amp.



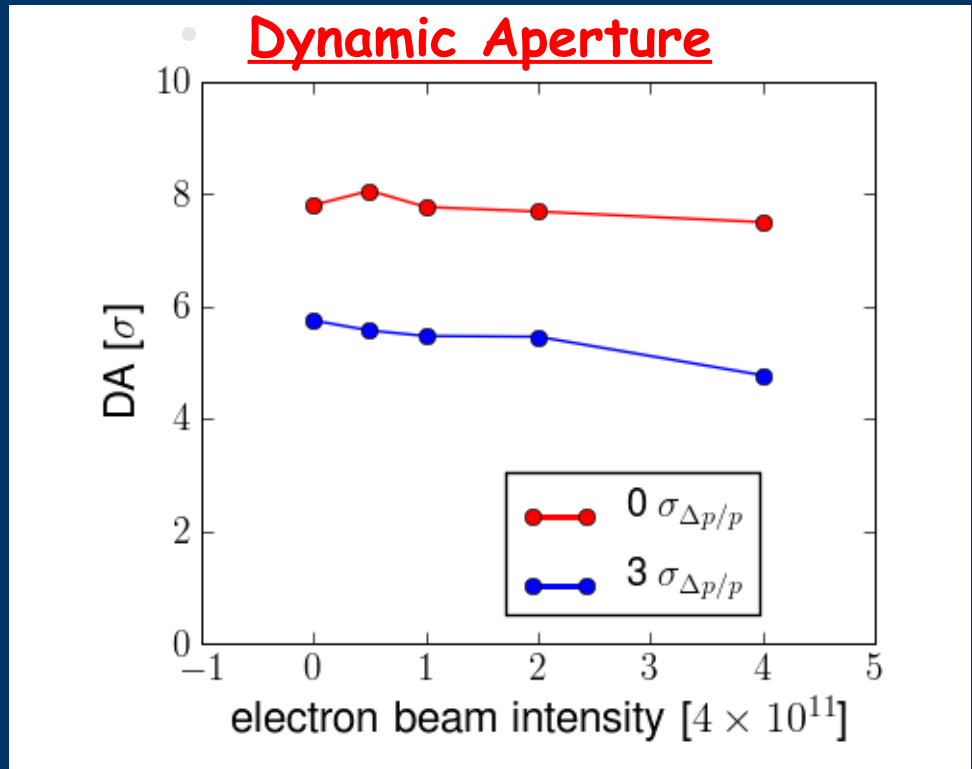
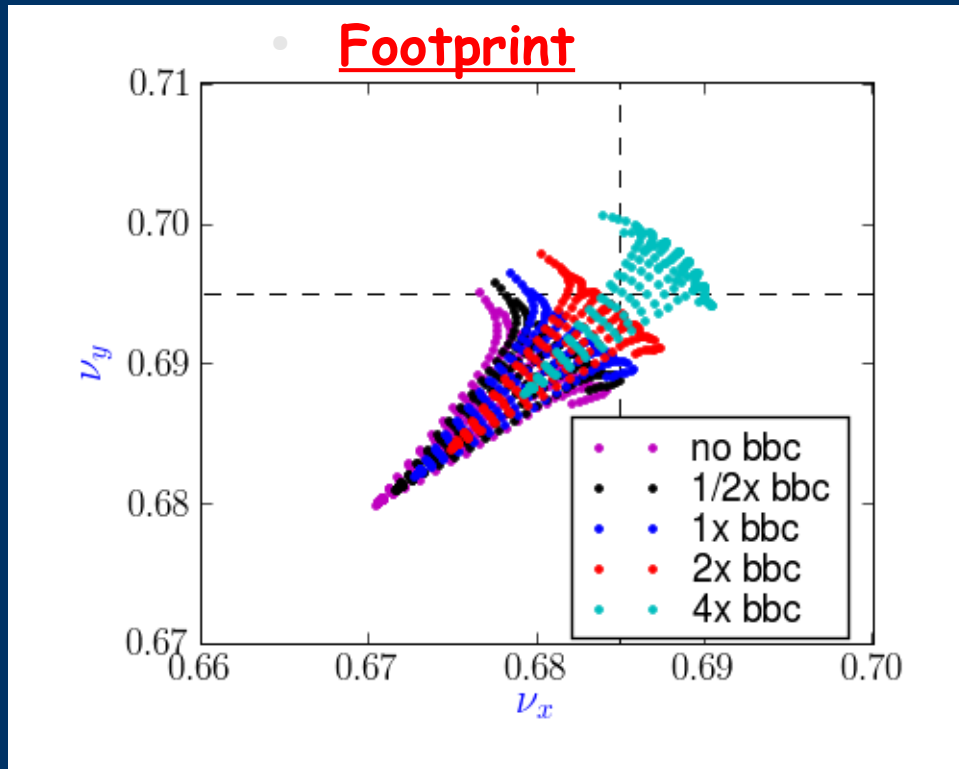
## Gaussian Electron Lens ( 2 sigma)

- Particle loss



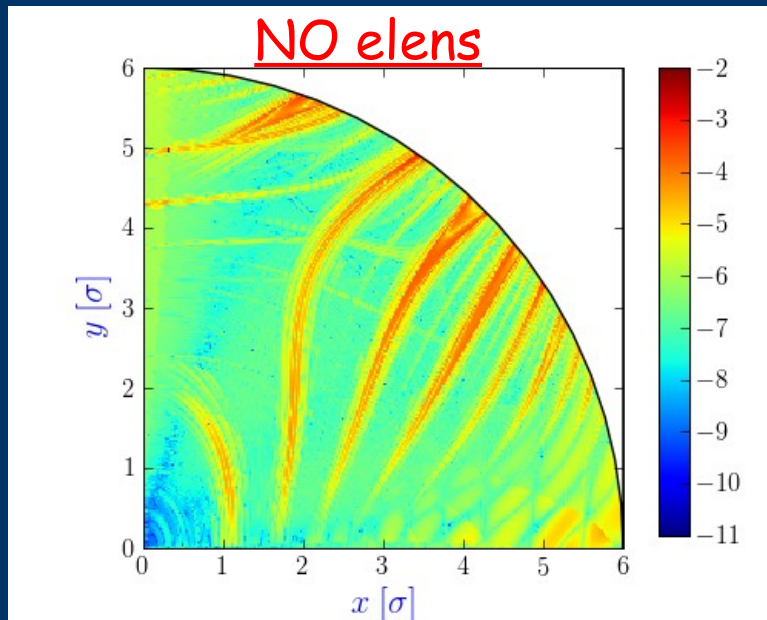
- Small Ne reduces beam loss:
  - $(\text{loss with } 1/2x \text{ bbc}) / (\text{loss with NO bbc}) \sim 10\%$

## SEFT Electron Lens ( 4 sigma)



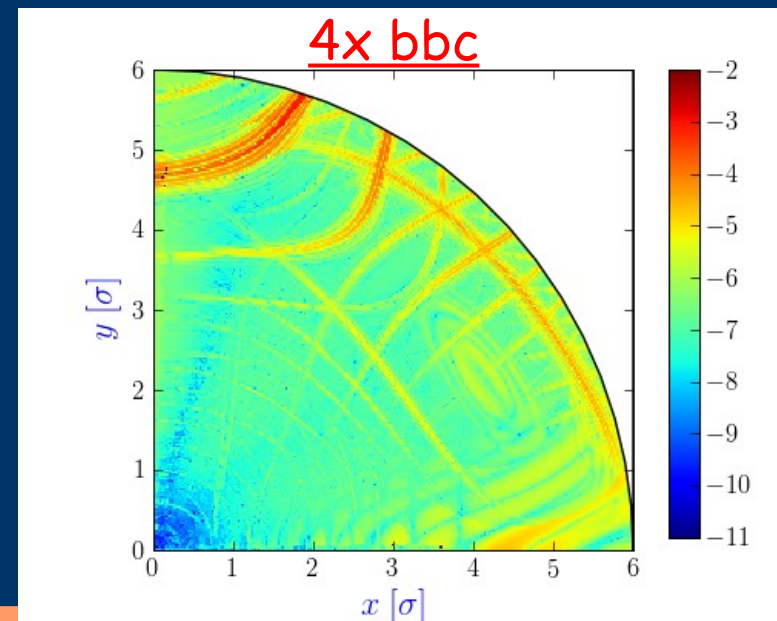
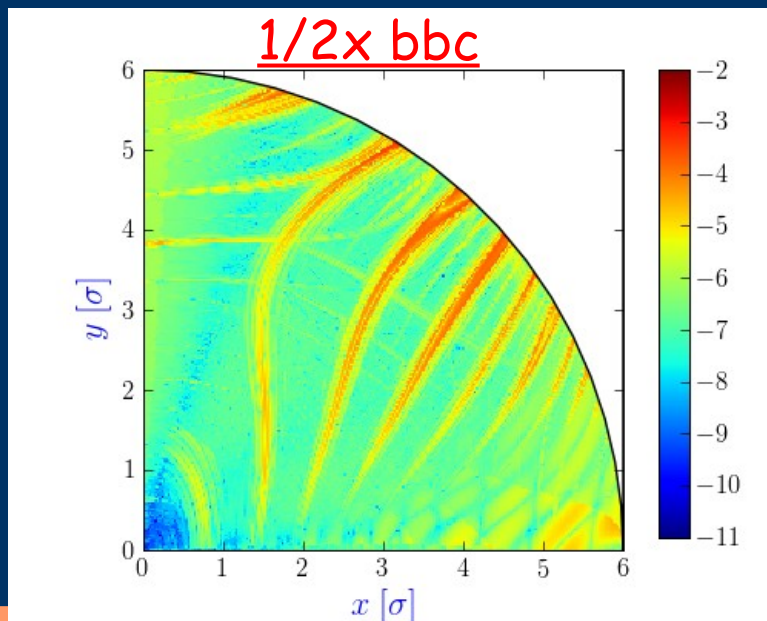
- Shape of footprint with compensation is almost the same as without compensation.
- Dynamic aperture is almost the same up to 2x bbc.

## SEFT Electron Lens ( 4 sigma)



- Tune Diffusion

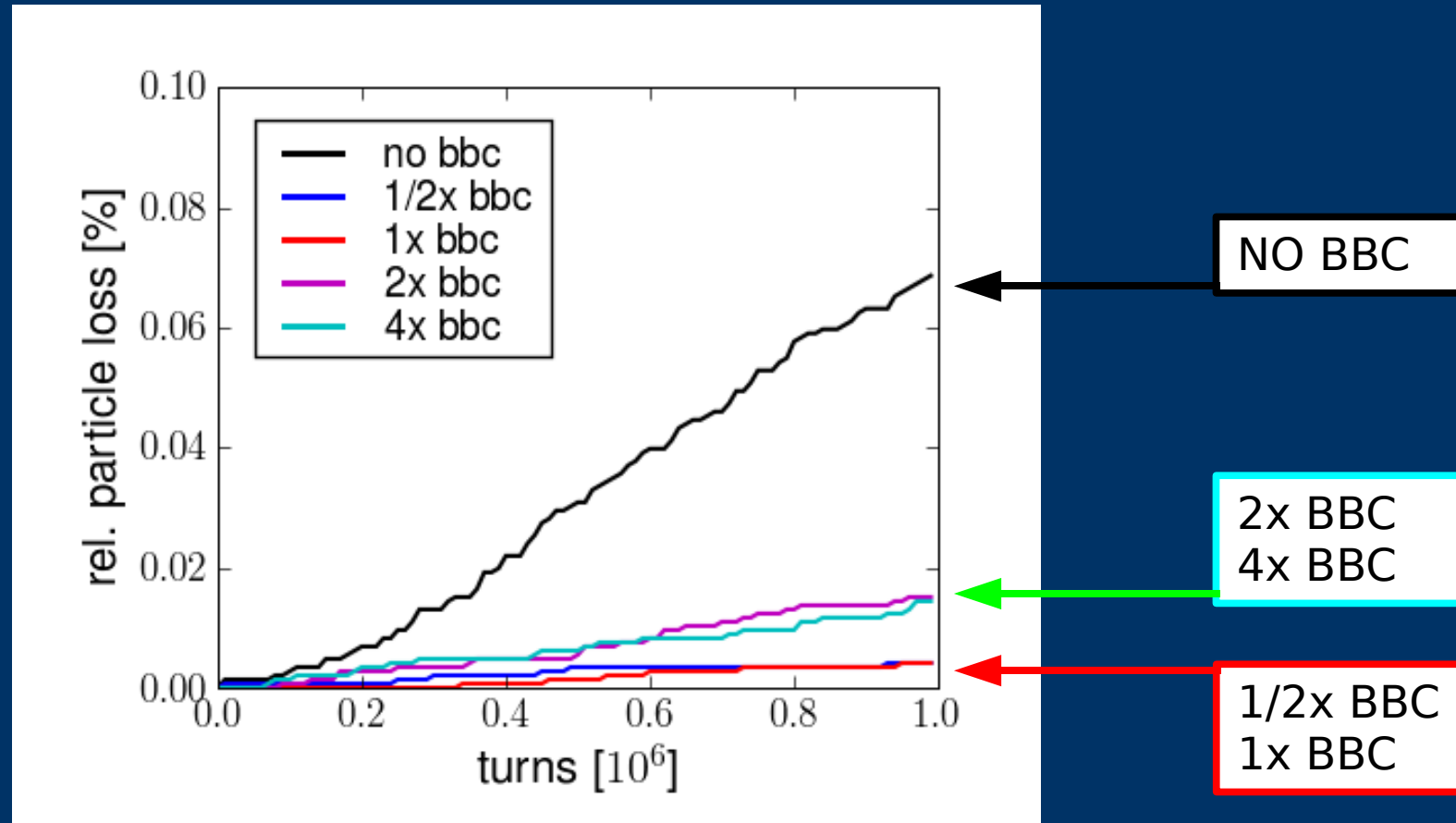
- 4x bbc: decrease tune change at small amp. but increase at large amp.
- 1/2x bbc: decrease tune change at both small and large amp.





## SEFT Electron Lens ( 4 sigma)

- Particle loss



- Small Ne reduces beam loss:
  - (loss of 1/2x bbc)/(loss of NO bbc)  $\sim 10\%$